BOARD OF STUDIES
NEW SOUTH WALES

2001
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 2

Revised Edition

General Instructions

- Panicking time – 5 minutes
- Scribbling furiously time – 3 hours
- Write using black or blue pen, any deviation will score 0
- Board-approved calculators may be used as a crutch
- A table of standard integrals is provided at the back of this paper as a security blanket
- All necessary working should be shown in every question, this includes defining the field laws each time number(s) of any field F are used

Total marks – 120

- Half-heartedly attempt questions 1-8, especially 8
- All questions are of equal value, but some are more equal than others

Total marks – 120 (dream on)
Attempt Questions 1-8
All questions are of equal value, but some are more equal than others

Answer each question in a SEPARATE writing booklet. That way when you only answer three questions it will still look impressive, and those trees were just lying about anyway. Extra writing booklets are available, but you won't be needing them.

Question 1 (15 marks) Use a SEPARATE writing booklet. Separate to what, we don't know.

[Note that due to HSC – sorry, New HSC – reforms, actual integration has been replaced by general knowledge. The terminology, however, remains.]

(a) Find the integral need to write an essay, from 0 to 40 minutes.

(b) By completing the squares, find how nerds there needs to be for you to be pushed from your ideal UAI by 0.05 points.

(c) Use integrity by past (that is, hindsight) to evaluate points scored from announcing a series of exams a success before the first one is completed.

(d) Use the substitution of $m = s$ to evaluate the number that defines who is suitable to attend university, where m = memory and s = social skills.

(e) (i) Find real people a and b such that

a works hard in a Humanities subject and is recognised for it and b bludges in this subject and is treated correspondingly.

(ii) Find the integrity of an independent body that insists, "We are not out to trick you."

Question 2 (15 marks) Use a SEPARATE writing booklet, idiot!

(a) Let z and w be complex issues such that z corresponds to education and w to insightful political debate. $\ddot{w}$ is the complex conjugate of w and corresponds to party bickering.

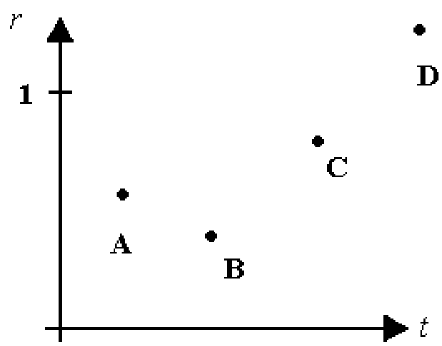
Prove that $z\ w = \ddot{w}$ and that w has no real component.

(b) (i) Express a complex debate in redneck-soundbite form.

(ii) Hence evaluate the margin that the next government will win by.

(c) Sketch a plan in the complex plane where students in different states are examined equally and fairly. The plan should be where $s + f$ is positive and should not be based entirely in the imaginary domain, where s represents state government and f represents federal.

(d) Find all the solutions to the "boat people crisis". Give your answers in redneck-soundbite form.



(e)

In the diagram the points A B and C show the various positions of three students on a plot of *results* against *time*. The students are private school, public school and selective school respectively.

(i) Explain why the representation of A, B and C is complex and not fully represented on this graph. Calculate how many dimensions would be required for an accurate representation.

(ii) Suppose D is the student such that they get the UAI they want. Find the complex equation representing how this is possible and the number of weeks they spend doing non-HSC things (as a fraction).

Question 3 (15 marks) Use a SEPARATE writing booklet, haven't you learned yet?

(a) Consider the hyperbole H that is ejected from a politician during an election campaign.

(i) Find the points of intersection of H with reality (the real plane), the eccentricity of the hyperbole and the lines H tends to. National security is one such line at the moment.

(ii) Sketch a cartoon of H.

(b) Numbers α, β and γ satisfy the equations

$$\alpha + \beta + \gamma = 42$$

$$\alpha^2 + \beta^2 + \gamma^2 = 2{,}000{,}000$$

$$\alpha\beta\gamma = 5{,}000$$

α = number of HSC exams
β = number of HSC supervisors watching the exams
γ = number of Tic Tacs each supervisor eats

(i) Find the values α and β if each student can stand only 12 Tic Tacs being rattled.

Explain why this is annoying, to your supervisor.

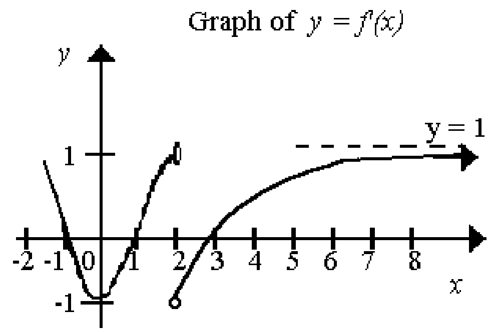
(ii) Find the minimum value for γ if each student is to stay sane, and each student sits 7 exams with 28 supervisors at each, and will turn postal when the supervisors break out in "Hot Shoe Shuffle."

(c) The area under your examination centre is infested with chirping insects between t=9 and t=17.

Use the method of calling an exterminator at BOS to find how many exam marks you can scam for this disruption.

Question 4 (getting tough now) Use a SEPARATE writing booklet. We want a new pen, too.

(a)

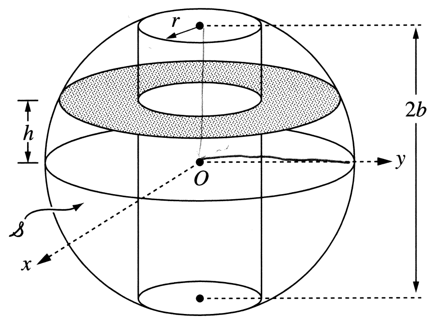


The diagram shows a sketch of $y = f'(x)$, the projected budget surplus for the next political term. The curve $y = f'(x)$ has a horizontal asymptote y = 1.

(i) Identify and classify along partisan lines any holes in the budget.

(ii) Sketch the curve of the actual budget once the election is over, given that f(x) may not be continuous and may contain imaginary features. On your diagram, clearly identify and label any important features including tending to 0. Colour any f(x) < 0 red, and repeatedly highlight any f(x) > f(0) where f(0) is the current surplus.

(b)



A cylindrical hole of radius *r* is bored through a sphere of radius *R*. The sphere represents your internal musings during a day and the cylinder represents the looming exams boring a hole through any non-HSC thoughts. The hole is initially small and in proportion to its importance and *r* increases as *t* does. To the observer *t* appears to shrink as it progresses and this gives the illusion that *R* is shrinking to fit *r*. The cross-section of & shown is a slice of your thoughts on any Monday.

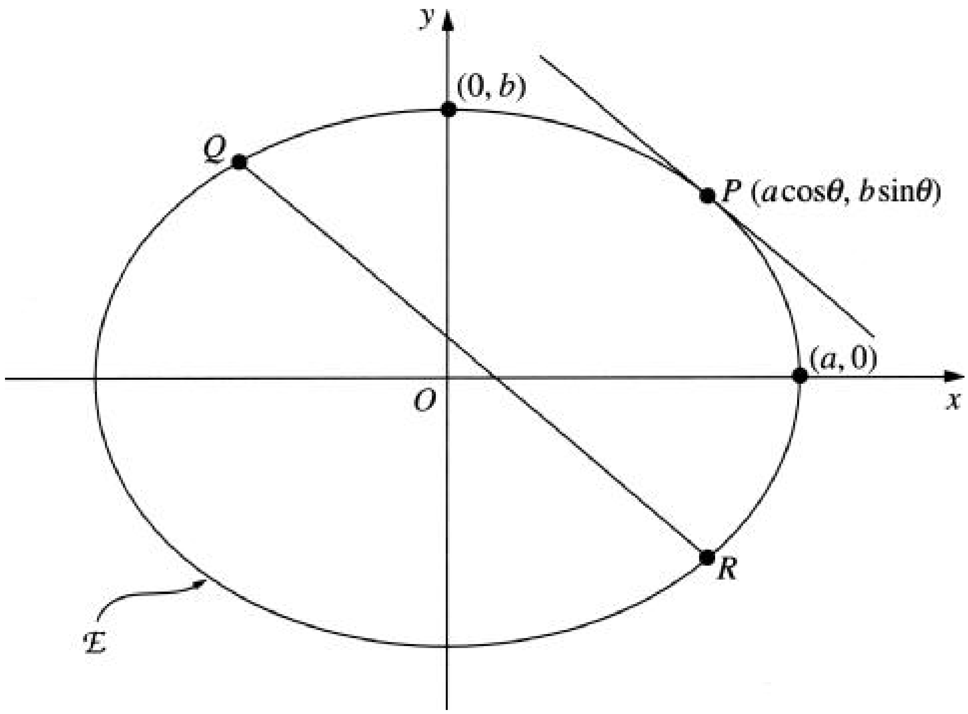
(i) Show that after the $t_{critical}$ is reached, *r* immediately shrinks and *R* becomes boundless.

(ii) Find the reason why the cross section grows increasingly dark as *t* approaches $t_{critical}$. In your answer refer to external factors and possibly the value of *b* and why it is even on this diagram.

(c) Use differentiation to show that the two major political parties are different. You may refer to doublethink and party acronym differences in order to avoid a null result.

Question 5 (15 marks) Use a SEPARATE writing booklet. If you can integrate surely you can follow a single instruction, IDIOT!

(a)



Consider the ellipse E, which encloses the area representing h, higher education. Each point on E represents a HSC course. The chord QR is an emphasis on technical knowledge. P is English.

(i) Show that Q and R represent Mathematics exams. Name these, and two other possible points of intersection.

(ii) Show that as h progresses, the chord PR progresses parallel to the tangent at P.

(iii) Deduce that even though a chord OP appears to intersect QR, the two lines do not actually meet in the real plane.

Question 5 continues on page 7

Question 5 (continued, duh, it was just a page ago!)

(b) An education minister of mass m is being held underwater by a submarine at maximum power. At maximum power, its engine blades deliver a force F on the minister's limbs. The sharks biting the limbs exert a resistive force proportional to the square of the amplitude of the minister's screams s.

i) Explain why

$$\lim_{t \to 0} L = 0$$

where L is the length of time the minister will live for after the piranhas also begin attacking.

ii) The submarine increases the speed of its blades from v1 to v2. Show that the distance the minister's screams will be heard underwater is m/2k times the distance his blood will spurt.

(c) A class of 22 students is to be divided into four groups consisting of 4, 5, 6 and 7 students.

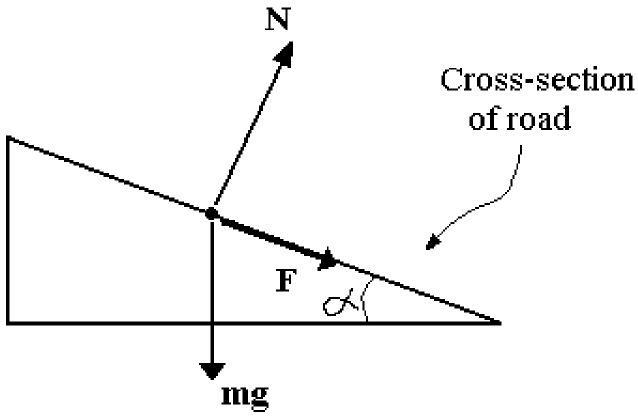
(i) In how many ways can this be done if the groups formed are stereotypes from American teen films? Leave your answer in unsimplified form because we don't really want you to have a reason to use your calculator.

(ii) Suppose that the four groups have been chosen.
In how many ways can the 22 students be arranged around a circular end-of-year table if the students of one group refuse to sit with another group? Name the number of possible summer romances if each nerd (all but one of whom are male) is attracted each female Beautiful Person (half are female). Name the number of combinations of jocks, loners and squares that can make up three possible movie leads, if one from each group must be represented.

End of question 5

Question 6 (another 15 marks down the drain) Use a SEPARATE writing booklet. When will you people learn?

(a)



A road contains a bend that is part of circle of radius r. At the bend, the road is banked at an angle α to the horizontal. A car travels around the bend at constant acceleration v (bet you didn't see that one coming). Assume the car is represented by a point of mass m and is driven by a teenage P-plater with percentage road sense approaching their shoe size. As the teenager does a burn-out, the forces acting on the car are the gravitational force mg, a sideways friction force F (acting down the road as drawn) and a normal reaction N to the road.

(i) By resolving never to get in a car with such an idiot, by how much have you increased your chances of surviving until next week?

(ii) Show that the mark on the road increases as F does.

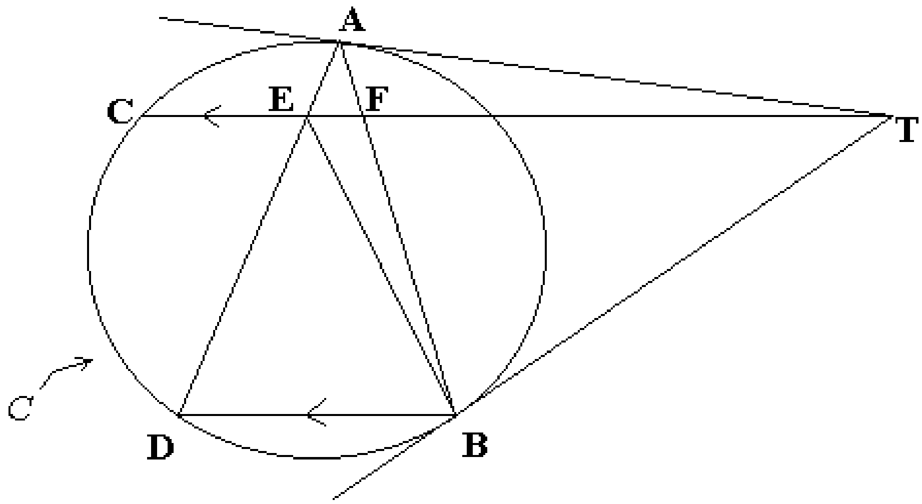
(iii) Suppose that the radius of the bend is 200m and that the road is banked to allow cars to travel at 100 kilometres per hour with no sideways friction force. Assume that the value of g is 9.8 ms^{-2}.

Find the probability that the driver lives to brag about the event. Assume the car is travelling at the usual speed of P-platers.

Question 6 continues on page 9

Question 6 (continued)

(b)



In the diagram, *C* is a circle with exterior point *T*. From *T*, tangents are drawn to the points *A* and *B* on *C* and a line *TC* is drawn, meeting the circle at *C*. The point *D* is the point on *C* such that *BD* is parallel to *TC*. The line *TC* cuts the line *AB* at *F* and the line *AD* at *E*. All of which is completely obvious from the diagram, but we just thought we'd make you read how we drew this pretty diagram anyway.

Copy or trace the diagram into your writing booklet. Bet yours is inferior!

(i) Prove that $\Delta FAT \equiv \Delta BAD$. This question was sponsored by the Jenny Craig Foundation.

(ii) Deduce that $DE.BT = TB^2$.

(iii) Show that ΔCAD is similar to ΔACT, or at least not entirely unfamiliar with it.

(iv) Prove that CAT ATE BAT, BAD BET. Express the odds of this happening in ratio form.

End of Question 6

Question 7 (15 marks, be nice wouldn't it?) Use a SEPARATE writing booklet, for the love of God!

(a) Suppose that z is a complex number, representing the time spent on the internet by an average HSC student each week, where the argument is that work is being done.

(i) Find the absolute truth of this.

(ii) Show that the imaginary part of this is the achievement of any actual work.

(iii) Find an expression for the number of online friends the average student can make, in terms of time spent in chat rooms and on message boards.

(b) Consider the equation $x3 - 3x - 1 = y$, which we denote by (*****EQUATION*****).

(i) Let $x = p/q$ where p and q are integers have no common divisors other than +1 and –1. Suppose that x is a root of the $ax3 - 3x + b = 0$, where a and b are integers. Assume that the commutative law for integers no longer applies and instead apply the alternative law as developed by Gauss. Assume that p and q are not relatively prime and that a and b are.

Explain why p divides b and why q divides a. Also explain why each of the number fields developed and state the inverse law for each such field. Represent a,b,p and q on a 4 dimensional graph in terms of x and y. Deduce that (*****EQUATION*****) doesn't really have anything to do with all these abstract statements.

(ii) Suppose that r, s and d are rational numbers and that $\sqrt{d}$ is irrational. Assume that $r + s\sqrt{d}$ is a root of (*****EQUATION*****).

Show that $3r2s + s3d - 3s = 0$ and show that $r - s\sqrt{d}$ must also be a root of (*****EQUATION*****).

Deduce from this result and part (I), that no root of (*****EQUATION*****) can be expressed in the form $r + s\sqrt{d}$ with r, s, and d rational.

Alternatively you may assume this question is irrationally long for 4 marks, laugh hysterically at the notion of completing it, and go to part (c).

(iii) Show that one root of (*****EQUATION*****) is x. You may assume this is so.

Question 8 (15 marks) Use a SEPARATE writing booklet. When will you ever learn?

(a) (i) Show that b rises exponentially as t approaches $t_{critical}$, where b is the number of babies kissed by any given politician and $t_{critical}$ is election time.

Hence deduce that any b should have $d \geq 50$m where d is the proximity of a politician.

(ii) Suppose that *a, b* and *c* are the sides of a triangle. Explain why. Deduce that the triangle exists.

(b) (i) Explain why, for a marginal electorate,

$$M \rightarrow \infty \text{ as } t \rightarrow t_{critical} \text{ as in (a)(i)}$$

where M is the result of copious pork-barrelling.

(You may assume the integral value of M after $t_{critical}$ is 0.)

(ii) Show, by induction, that there exist integers.

(iii) Suppose that integers do exist, a fact of which you are rationally positive. Show that for integers a and b they exist only the ideal Platonic realm. Represent these integers exactly on a two dimensional graph.

(iv) Prove the four-colour map theorem.

Just kidding, that would be really mean. Okay, prove e is irrational instead.

End of paper (now go back and finish all those parts you skipped)