ELEC2041

Microprocessors and Interfacing

Lectures 22 : Floating Point Number Representation – III

http://webct.edtec.unsw.edu.au/

April 2005

Saeid Nooshabadi

saeid@unsw.edu.au

Overview

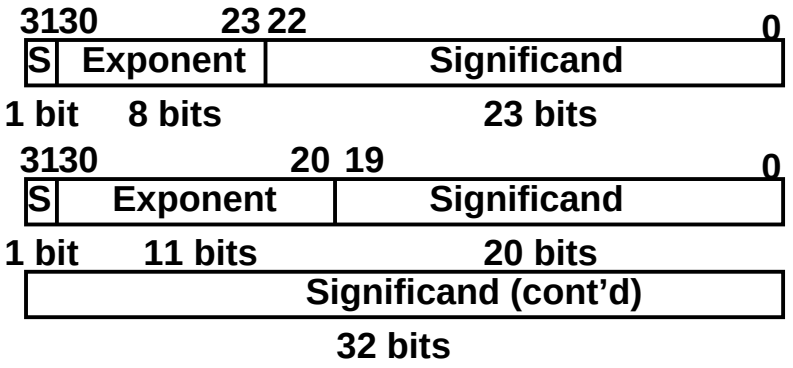
- Special Floating Point Numbers: NaN, Denorms
- IEEE Rounding modes
- Floating Point fallacies, hacks
- Using floating point in C and ARM
- Multi Dimensional Array layouts

Review: ARM Fl. Pt. Architecture

- Floating Point Data: approximate representation of very large or very small numbers in 32-bits or 64-bits
- IEEE 754 Floating Point Standard is most widely accepted attempt to standardize interpretation of such numbers
- New ARM registers(s0-s31), instruct.:
  - Single Precision (32 bits, $2x10^{-38}$… $2x10^{38}$): `fcmps, fadds, fsubs, fmuls, fdivs`
  - Double Precision (64 bits , $2x10^{-308}$…$2x10^{308}$): `fcmpd, faddd, fsubd, fmuld, fdivd`
- **Big Idea: Instructions determine meaning of data; nothing inherent inside the data**

Review: Floating Point Representation

- Single Precision and Double Precision



- $(-1)^{S} \times (1+\text{Significand}) \times 2^{(\text{Exponent}-\text{Bias})}$

New ARM arithmetic instructions

Example	Meaning	Comments
fadds s0,s1,s2	s0=s1+s2	Fl. Pt. Add (single)
faddd d0,d1,d2	d0=d1+d2	Fl. Pt. Add (double)
fsubs s0,s1,s2	s0=s1 – s2	Fl. Pt. Sub (single)
fsubd d0,d1,d2	d0=d1 – d2	Fl. Pt. Sub (double)
fmuls s0,s1,s2	s0=s1 × s2	Fl. Pt. Mul (single)
fmuld d0,d1,d2	d0=d1 × d2	Fl. Pt. Mul (double)
fdivs s0,s1,s2	s0=s1 ÷ s2	Fl. Pt. Div (single)
fdivd d0,d1,d2	d0=d1 ÷ d2	Fl. Pt. Div (double)
fcmps s0,s1	FCPSR flags = s0 – s1	Fl. Pt.Compare (single)
fcmpd d0,d1	FCPSR flags = d0 – d1	Fl. Pt.Compare (double)

Z = 1 if s0 = s1, (d0 = d1)
N = 1 if s0 < s1, (d0 < d1)
C = 1 if s0 = s1, (d0 = d1); s0 > s1, (d0 > d1), or unordered
V = 1 if unordered

Unordered? Next slide

Special Numbers

- What have we defined so far? (Single Precision)

Exponent	Significand	Object
0	0	0
0	nonzero	???
1-254	anything	+/- fl. pt. #
255	0	+/- infinity
255	nonzero	???

- Professor Kahan had clever ideas; "Waste not, want not"

Representation for Not a Number

- What do I get if I calculate `sqrt(-4.0)` or `0/0`?
  - If infinity is not an error, these shouldn't be either.
  - Called Not a Number (NaN)
  - Exponent = 255, Significand nonzero
- Why is this useful?
  - Hope NaNs help with debugging?
  - They contaminate: op(NaN,X) = NaN
  - OK if calculate but don't use it
  - `cmp s1, s2` produces unordered results if either is an NaN

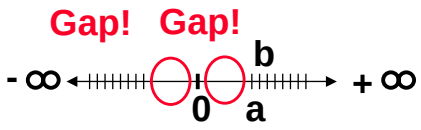
Special Numbers (cont'd)

- What have we defined so far? (Single Precision)?

Exponent	Significand	Object
0	0	0
0	nonzero	???
1-254	anything	+/- fl. pt. #
255	0	+/- infinity
255	nonzero	NaN

Representation for Denorms (#1/2)

- Problem: There's a gap among representable FP numbers around 0
  - Significand = 0, Exp = 0 (2^{-127}) ➔ 0
  - Smallest representable positive num:
    - $a = 1.0\ldots_2 * 2^{-126} = 2^{-126}$
  - Second smallest representable positive num:
    - $b = 1.000\ldots\ldots1_2 * 2^{-126} = 2^{-126} + 2^{-149}$
  - $a - 0 = 2^{-126}$
  - $b - a = 2^{-149}$



Representation for Denorms (#2/2)

- Solution:
  - We still haven't used Exponent = 0, Significand nonzero
  - Denormalized number: no leading 1
  - Smallest representable pos num:
    - $a = 2^{-149}$
  - Second smallest representable pos num:
    - $b = 2^{-148}$

Meaning: $(-1)^S$ x (0 + Significand) x $2^{(-126)}$

Range: $2^{-149} \le X \le 2^{-126} - 2^{-149}$

Special Numbers

- What have we defined so far? (Single Precision)

Exponent	Significand	Object
0	0	0
0	nonzero	Denorm
1-254	anything	+/- fl. pt. #
255	0	+/- infinity
255	nonzero	NaN

- Professor Kahan had clever ideas; "Waste not, want not"

Clever Idea and Hardware Implementation

- 0 nonzero Denorm
  - Very Clever Idea by Prof Kahan
  - BUT such corner cases make the hardware design very complex
  - Good idea but hard practice!
  - Even software emulation not easy.
  - In Ref. Ft. Pt. Emulator: A student mini project on:
    http://dsl.ee.unsw.edu.au/unsw/projects/armvfp/README.html

25% - 30% of the code is to get the operations on denorms right

- In most hardware implementations denorms are flushed to zero, or implemented in software via exceptions

Rounding

- When we perform math on real numbers, we have to worry about rounding
- The actual hardware for Floating Point Representation carries two extra bits of precision, and then round to get the proper value
- Rounding also occurs when converting a double to a single precision value, or converting a floating point number to an integer

IEEE Rounding Modes

- Round towards +infinity
  - ALWAYS round "up": 2.2001 → 2.3
  - -2.3001 → -2.3
- Round towards -infinity
  - ALWAYS round "down": 1.9999 →1.9,
  - -1.9999 → -2.0
- Truncate
  - Just drop the last digits (round towards 0); 1.9999 → 1.9, -1.9999 → -1.9
- Round to (nearest) even
  - Normal rounding, almost

Round to Even

- Round like you learned in high school
- Except if the value is right on the borderline, in which case we round to the nearest EVEN number
  - 2.55 → 2.6
  - 3.45 → 3.4
- Insures fairness on calculation
  - This way, half the time we round up on tie, the other half time we round down
  - Ask statistics Prof.
- This is the default rounding mode

Casting floats to ints and vice versa in C

- `(int)` *exp*
  - In C `float` to `int` type casting coerces and converts it to the an integer by truncation (rounds to towards 0)
  - affected by rounding modes
  - `i = (int) (3.14159 * f);`
  - `fuitos` (floating → int) In ARM to round to a selected mode (default nearest)
  - `fuitozs` (floating → int) In ARM to round towards zero
- `(float)` *exp*
  - converts integer to nearest floating point
  - `f = f + (float) i;`
  - `fsitos` (int → floating) In ARM

int → float → int

```
if (i == (int)((float) i)) {
 printf("true");
}
```

- Will not always work
- Large values of integers don't have exact floating point representations
- Similarly, we may round to the wrong value

float → int → float

```
if (f == (float)((int) f)) {
 printf("true");
}
```

- Will not always work
- Small values of floating point don't have good integer representations
- Also rounding errors

Ints, Fractions and rounding in C

- What do you get?

```
{ int x = 3/2;      int y = 2/3;
  printf("x: %d, y: %d", x, y); }
```

- How about?

```
int cela =((fahr - 32) * 5)/ 9;
int celb = (5 / 9) * (fahr - 32)
float celc = (5.0 / 9.0) * (fahr - 32);
```

fahr = 60 =>
cela: 15,
celb: 0,
celc: 15.55556

Floating Point Fallacy

- FP Add, subtract associative: FALSE!

  X +(Y + z) = (X + y) + Z

  - x = – 1.5 x 10^{38}, y = 1.5 x 10^{38}, and z = 1.0
  - x + (y + z) = –1.5x10^{38} + (1.5x10^{38} + 1.0)
    = –1.5x10^{38} + (1.5x10^{38}) = 0.0
  - (x + y) + z = (–1.5x10^{38} + 1.5x10^{38}) + 1.0
    = (0.0) + 1.0 = 1.0

- Therefore, Floating Point add, subtract are not associative!
  - Why? FP result approximates real result!
  - In this example: 1.5 x 10^{38} is so much larger than 1.0 that 1.5 x 10^{38} + 1.0 in floating point representation is still 1.5 x 10^{38}

Floating Point In the News!

- July 1994: Intel discovers bug in Pentium
  - Occasionally affects bits 12-52 of D.P. divide
  - The bug was introduces when they optimsed divide unit to run much faster. They ignored some rare corner cases
- Sept: Math Prof. discovers, puts on WWW
- Nov: Front page trade paper, then NY Times
  - Intel: "several dozen people that this would affect. So far, we've only heard from one."
  - Intel claims customers see 1 error/27000 years for random set of Ft. Pt. Inputs.
  - Does not explain why anybody wants to use Ff. Pt. No. in random
  - IBM claims 1 error/month, stops shipping
- Dec: Intel apologizes, replace chips $300M

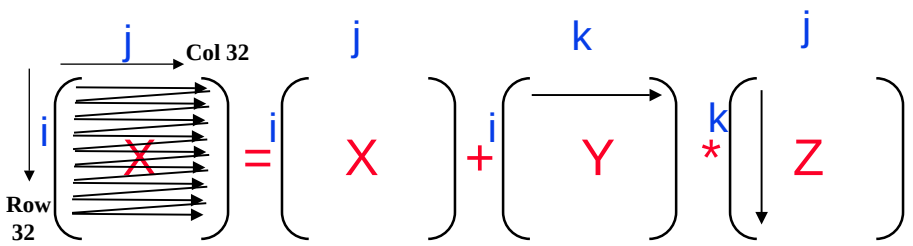
IEEE 754 Floating Point Issues

- It is complex, involves lots of details
  - We just scratched the surface
- Check for gradual underflow and treating denomrs makes it much harder
- Beyond Prof. Kahan very few really understand it!
- It was finally approved as IEEE 754 after 10 years of controversy in 1983
  - Denorm was the most controversial aspect
  - The visitors to the US were advised of 3 most interesting places to visit: Las Vegas, Great Canyon and IEEE committee rooms!

Reading Material

- ARM Architecture Reference Manual 2nd Ed, Addison-Wesley, 2001, ISBN: 0-201-73719-1, Part C, Vector Floating Point Architecture, chapters C1 – C5
- Ft. Pt. Emulator: A student mini project on: http://dsl.ee.unsw.edu.au/unsw/projects/armvfp/README.html
- Steve Furber: ARM System On-Chip; 2nd Ed, Addison-Wesley, 2000, ISBN: 0-201-67519-6. chapter 6 (NOT up to date)

Example: Matrix with Fl Pt, Multiply, Add?



Example: Matrix with Fl Pt, Multiply, Add in C

```
void mm(double x[][32],double
 y[][32], double z[][32]){
 int i, j, k;

 for (i=0; i<32; i=i+1)
   for (j=0; j<32; j=j+1)
     for (k=0; k<32; k=k+1)
       x[i][j] = x[i][j] +
                 y[i][k] * z[k][j];
}
```

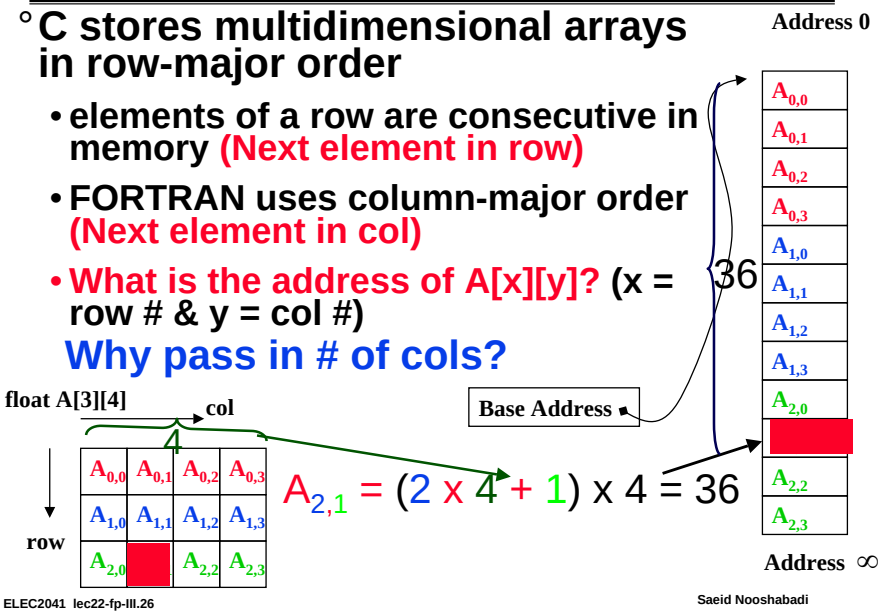
Why pass in # of cols?

- Starting addresses are parameters in a1, a2, and a3. Integer variables are in v2, v3, v4. Arrays 32 x 32
- Use `fldd`/`fstd` (load/store 64 bits)

Multidimensional Array Addressing

- C stores multidimensional arrays in row-major order
  - elements of a row are consecutive in memory (Next element in row)
  - FORTRAN uses column-major order (Next element in col)
  - What is the address of A[x][y]? (x = row # & y = col #)

Why pass in # of cols?



ARM code for first piece: initilialize, x[][]

- Initailize Loop Variables

```
mm: ...
    stmfd sp!, {v1-v4}
    mov v1, #32  ; v1 = 32
    mov v2, #0   ; i = 0; 1st loop
L1: mov v3, #0   ; j = 0; reset 2nd
L2: mov v4, #0   ; k = 0; reset 3rd
```

- To fetch x[i][j], skip i rows (i*32), add j

```
    add  a4,v3,v2, lsl #5 ;a4 = i*2^5+j
```

Get byte address (8 bytes), load x[i][j]

```
    add  a4,a1,a4, lsl #3;a4 = a1 +a4*8
                          ;(i,j byte addr.)
    fldd d0, [a4]         ; d0 = x[i][j]
```


ARM code for second piece: z[][], y[][]

- Like before, but load y[i][k] into d1

```
L3: add  ip,v4,v2, lsl #5 ;ip = i*2^5+k
    add  ip,a2,ip, lsl #3 ;ip = a2 +ip*8
                          ;(i,k byte addr.)
    fldd d1, [ip]         ; d1 = y[i][k]
```

- Like before, but load z[k][j] into d2

```
    add  ip,v3,v4, lsl #5 ;ip = k*2^5+j
    add  ip,a3,ip, lsl #3 ;ip = a3 +ip*8
                          ;(k,j byte addr.)
    fldd d2, [ip]         ; d2 = z[k][j]
```

- Summary: d0:x[i][j], d1:y[i][k], d2:z[k][j]

ARM code for last piece: add/mul, loops

- Add y*z to x

```
fmacd d0,d1,d2    ; x[][] = x + y*z
```

- Increment k; if end of inner loop, store x

```
add v4,v4,#1      ; k = k + 1
cmp   v4,v1       ; if(k<32) goto L3
blt   L3
fstd d0,[a4]      ; x[i][j] = d0
```

- Increment j; middle loop if not end of j

```
add v3,v3,#1      ; j = j + 1
cmp   v3,v1       ; if(j<32) goto L2
blt   L2
```

- Increment i; if end of outer loop, return

```
add   v2,v2,#1    ; i = i + 1
cmp   v2,v1       ; if(i<32) goto L1
blt   L1
```


ARM code for Return

- Return

```
ldmfd sp!, {v1-v4}
mov pc, lr
```


"And in Conclusion.."

- Exponent = 255, Significand nonzero Represents NaN
- Finite precision means we have to cope with round off error (arithmetic with inexact values) and truncation error (large values overwhelming small ones).
- In NaN representation of Ft. Pt. Exponent = 255 and Significand ≠ 0
- In Denorm representation of Ft. Pt. Exponent = 0 and Significand ≠ 0
- In Denorm representation of Ft. Pt. numbers there no hidden 1.