Signal Separation using Time Frequency Representation

Viswanath Ganapathy, Ranjeet K. Patro, Chandrasekhara Thejaswi, Manik Raina, Subhas K. Ghosh
Honeywell Technology Solutions Laboratory
151/1, Doraisanipalya, Bannerghatta Road,
Bangalore, India, 560076
Email:viswanath.ganapathy@honeywell.com

I. Abstract

Signals which can be modeled as linear combination of polynomial phase signals appear in several applications including active noise cancellation, communication using chirp modulation, radar etc. In this paper we discuss an approach using optimal time frequency representation and hough transform to separate linear combination polynomial phase signals. Further we also show the performance of our approach using simulation results and the importance of choosing optimal time frequency representation for signal separation.

II. Introduction

In this paper we discuss an approach for separating the component signals from a linear combination of several signal. We assume, the signal model as a linear combination of polynomial phase signals, i.e. $y(t) = \sum_{k=1}^{k=K} a_k e^{i\sum_{j=0}^{n(j)} b_{kj} t^j}$. In general obtaining a_k and b_{kj} is a hard problem. Assuming that a_k, K and maximum degree of the polynomial phase are known, we discuss an approach to obtain the coefficients of the component polynomial phase signal. In this paper we show the performance of our approach using simulation results the importance of choosing optimal time frequency representation for signal separation. Before we discuss our approach we briefly discuss time frequency representation and hough transform.

Time frequency representation (TFR): Time-frequency representations of signals map a one-dimensional signal of time, $x(t)$, into a two dimensional function of time and frequency, $T_X(t, f)$. They thus combine time-domain and frequency-domain analysis to yield a potentially more revealing picture of the temporal localization of signal's spectral components. The well known examples of TFRs include the short time Fourier transform (STFT) [2] and Wigner-Ville distribution (WVD)[2]. WVD is a bilinear representation of a signal while STFT is linear. Therefore WVD introduces cross terms for multi component signals. These cross terms in WVD are removed using smoothing kernels. The choice of smoothing kernel gives rise to different TFRs. Some examples of smoothing kernels include the Born Jordan kernel and cone kernel representation (CKR). In this paper we use CKR for our simulation. The CKR [7] constrains the temporal smoothing so that the spectral peaks are enhanced. Thus, for a discrete sequence $x(n)$, assuming a symmetric window $\rho(n) \neq 0, -L \leq n \leq L, N = 2L + 1$ the discrete CKR can be expressed as

$$CKR(n, w) = \frac{1}{2L+1} \sum_k \sum_m \rho(k) x(m + \frac{k}{2}) x^*(m - \frac{k}{2}) \exp(-jwk)$$

where, k takes values from $-L$ to L and m takes values from $n - (\frac{|k|}{2})$ to $n + (\frac{|k|}{2})$. The cone kernel representation has several interesting properties [6]. These include noise robustness and location of cross terms on the auto terms.

Instantaneous Frequency (IF)[2]: The instantaneous frequency of a signal is defined as the derivative of the phase of the signal. The periodic first moment of a TFR can be used to estimate the IF a signal. IF estimate based on the first moment is the mean of the component frequencies at that instant.

Hough Transform[3]: The Hough transform(HT) is a standard tool in image analysis that allows recognition of global patterns in an image space by recognition of local patterns (ideally a point) in a transformed parameter space. The basic idea of this technique is to find curves that can be parameterized like straight lines, polynomials, circles, bessel functions etc., in a suitable parameter space. HT can also be used to detect patterns in higher dimensions. Consider an example of identifying sets of collinear points in an image. A set of image points (x, y) which lie on a straight line can be defined by a relation, f, such that $f((m, c), (x, y)) = y - mx - c = 0$, where m and c are slope and intercept respectively. The mapping from the space of possible parameter values to the set of (x, y) points is a one to many mapping. The HT of above equation can be seen as a mutual constraint between the set of points and the set of parameters. Therefore it can be interpreted as defining one to many mapping from a point (x, y) to set of possible parameter values or a point in transform space (m, c) to set of points in signal space. In the case of a straight line, all the points lie along a line will have the same value in the parameter space and therefore will intersect at a point in the parameter space. Determination of the point intersection is equivalent to picking the maximum value attained in the (m, c) space.

III. Our Approach

Our signal separation approach is based on choosing an optimal time frequency representation for the given signal and applying hough transform on the chosen optimal time frequency representation. The Wigner-Ville representation is known to be optimal time frequency representation for single component linear signals. For multi-component signals the reduced interference distribution using Born-Jordan kernel is optimal. In general for single component polynomial phase signal higher order distributions like polynomial Wigner-Ville distribution [1] or the L-Wigner distribution [5] is optimal. But for polynomial Wigner-Ville representation cross terms will introduce difficulty for higher order polynomial phase signal. The realization of polynomial Wigner-Ville representation for multi component signals is discussed in [4]. Therefore L-Wigner representation computed with a knowledge of signal separation in the time frequency domain will lead to an optimal TFR for polynomial phase signals (here we ensure that there are no cross terms). Moreover the dimension of the parameter space to which Hough transform maps the time frequency representation is also decided based on the greatest degree of the polynomial phase signal. In this paper, we show the performance of our approach using simulation results and the importance of choosing optimal time frequency representation for signal separation.

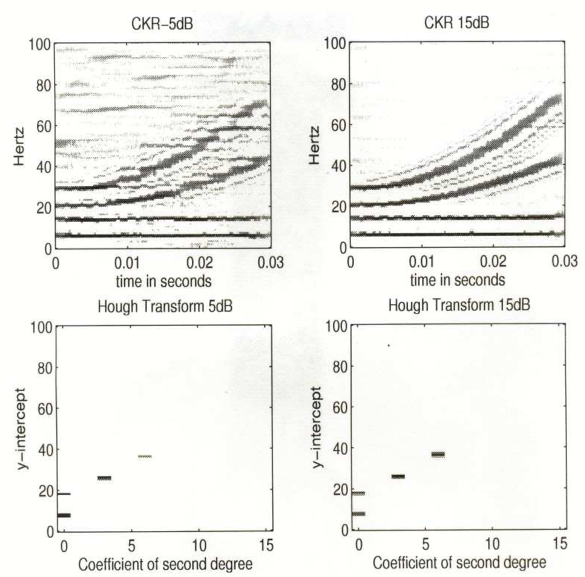

Fig. 1. CKR and associated hough transform of a four component signal. The top row right and top left windows show the CKR of four component signal at 5 dB and 15 dB SNR. The bottom right and left windows show the associated hough transform.

IV. Simulation and Discussion

We consider four component signal composed of sinusoids and cubic chirps. The CKR is computed and the hough transform of CKR is obtained. The CKR and hough transform of CKR is obtained for various SNRs ranging from 0dB to 10 dB. The hough transform of the CKR is computed by integrating in the TF domain along all possible paths in the TF domain. Here we use peaks in the hough transform domain as the initial estimates of the signal track and then look for peaks around the local region represented by these hough transform parameters in the TF domain. The hough transform of the four component signals is shown in figure 1. There are four peaks corresponding to the four signal tracks in the hough transform domain. These peaks give an initial estimate of the IF law of each component. The hough transform performance deteriorates around 6dB SNR, below which it fails to detect the cubic chirps. The sinusoids in the TFR domain is detected up to 3dB. Here it is important to note that the CKR is an optimal TFR for sinusoids alone. Therefore it is expected to perform better for sinusoids for the following reason. We detect the distinct signal components by identifying peaks in the Hough transform domain. If the signal tracks are represented by a delta function the number points in a particular bin (representing the parameter of the signal) increases when compared to the other bins in the parameter space. Therefore SNR seen in the hough transform domain is high for optimal TFRs. The detected peaks are used to represent the regions in the CKR domain and an IF estimate is made based on the local peaks. The resulting error in estimate i.e., both the mean and variance, is given for all the four signal components in Table I.

TABLE I

Mean(m) and Variance(v) in the IF estimates of four component signal using Hough Transform and CKR

SNR(dB)	First	Second	Third	Fourth
0	3.4765(m), 0.014(v)	7.3828(m), 0.014(v)	*(m), *(v)	*(m), *(v)
3	3.48(m), 0.014(v)	7.42(m), 0.014(v)	*(m), *(v)	*(m), *(v)
6	3.5156(m), 0(v)	7.42(m), 0(v)	13.164(m), 0.093(v)	20.43(m), 1.5(v)
7	3.5156(m), 0(v)	7.42(m), 0(v)	13.12(m), 0.07(v)	20.15(m), 0.19(v)

Note: The true value of IF are $3Hz$, $7Hz$, $13Hz$ and $20Hz$ at $0.015s$. Further, $*$ denotes those components that were not detected by Hough transform.

V. Conclusion and Future Work

In this paper we shown that the performance of separation multi-component using time frequency representation and highlighted the importance of using optimal TFR for the given signal. Further we have also given insights on the performance analysis. As a part of further work we are obtaining theoretical performance bounds for parameters estimated using TFR and hough transform. We are also working on comparing the performance of polynomial Wigner-Ville representation and L-Wigner representation for signal separation of multi component signals.

REFERENCES

[1] B.Boashash and O'shea, "Polynomial Wigner-Ville distribution and their relationship to time varying higher order spectra", *IEEE Transactions on Signal Processing*, 42, pp. 216-220, 1994.

[2] L.Cohen, " Time Frequency Analysis", *Eaglewood Cliffs, Prentice Hall*, NJ, Chapter 5, 1995.

[3] J.Illinworth and J.Kittler, "A Survey of Hough Transform", *Computer Vision Graphics and Image Processing*, 44, pp. 87-116, 1988.

[4] L.J.Stankovic, "On the Realization of the Polynomial WignerVille Distribution for Multicomponent Signals", *IEEE Signal Processing Letters*, Vol. 5, No. 7, July 1998.

[5] L.J.Stankovic, "L-Class of time-frequency distributions", *IEEE Signal Processing Letters*, 3, pp. 22-25, Jan 1996.

[6] G.Viswanath and T.V.Sreenivas, "Cone-kernel representation versus instantaneous power spectrum", *IEEE Transactions on Signal Processing*, 47(1), pp. 250-254, 1999.

[7] Y.Zhao, L.E.Atlas and R.J.Marks, "The use of cone shaped kernels for generalized time-frequency representations for non-stationary signals", *IEEE Transaction on Acoustic, Speech, Signal Processing*, 38, pp. 1084-1091, 1990.