$$\sqrt[2]{2}$$

gives us
"The Key to the Door."

Laurence M. Rash

$$\sqrt[2]{2}$$

gives us

"THE KEY TO THE DOOR".

Laurence M. Rash

July 15, 1968.

age II

al number

Union, Iowa
Oct. 8, 1971

Presented to my daughter, Nora Rosella, whom, I hope, will become my understudy.

Laurence M. Rash.

I dedicate this exposition of my theory to the memory of my late father, Monroe Rash, whose ability in arithmetic I greatly admired.

Laurence Monroe Rash.

Laurence M. Rash
Union Iowa

PREFACE

In this work I have tried to follow the trail of my own advancement in the solution of this problem. It might be that I could have explained it in fewer pages, if I had attacked each principle or fact more directly; but this was the approach that appealled to me. The dates affixed to each Section, are the dates upon which my written work was done, and approximately the date of discovery of each principle.

I — Essentially this method of extracting square root is just another method of extraction. As such it is an advance in pure mathematics. This method of extraction is based on the natural system of numbers, and uses common fractions. It delves deeply into the mystery of existence. As applied mathematics, this method uses large and complicated sets of numbers in its calculations. For that reason it is a more ponderous and unwieldy tool than the Binomial Theorem method. The only chance for this method to be faster would be with common fractions equivalent to decimals having a large number of digits.

In contrast, Sir Isaac Newton's development of the Binominal Theorem is based on the decimal system of numbers, and may give roots in decimal fractions. Because of this, it can be associated with logarithm tables, and lends itself to speedy results.

If pursued deligently, one method is as accurate as the other. Either method can be made to give results far beyond our capacity to use.

I believe the essential propositions of this (my) theory are completely proven.

II — Inherently this method opens the door to a study of the barriers between the difference dimensional continua, in the sense in which "continuum" is used when Albert Einstein speaks of Space as a three dimensional continuum, and also of a four dimensional space-time continuum — page 65 Relativity — The Special and the General Theory.

This (my) method does not approach the subject directly, but rather obliquely. In medicine the doctors prescribe specific remedies for the treatment of disease. Also there are " side effects". These insights into the nature of the barriers between dimensional continua are but incidental to this theory of solution of the roots. I might add that much of my progress in the solution of the problem has been collaterally. This is not all that might be wished, but I believe that the door is ajar.

I believe, what I consider Inherent propositions of my theory will be controversial.

III — I have other thoughts, only partly developed, on this subject..

Yours,

Laurence M. Rash
Jan. 1967.

Page V

Laurence M. Rash
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(Title) — a Guide Thru the Progressive Steps of this Theory, So that the Reader may Know of the Goal or Purpose of Each Section or Group of Sections of my Book.

Chapter I — Birth

Step 1. — Section 1. a — This is the "funny little table" which started my thoughts on this theory.

Section 1. b — The developement of the formula $(\text{inte})^2 = (\text{inte} - D)(\text{inte} + D) + D^2$.

Sections 2 and 3 prove that this formula is only a variation of the binomial formula.

Chapter II — Childhood.

Section 4. — There can be no eliminations between two variations of the same equation.

Section 5. — The difference between the squares of two quantities Page 3. b. gives the last difference* $(a^2 - n^2)$.

Step 2 — Page 3.c $(a^2 - n^2) = (2v - 1)d^2$. This is important because it is essential to "setting up" the diagrams.

Section 6, Page 3. c; also value of v explained and defined.

Section 7. — Explanation of tables of Section 6.

Section 8. — value of $(a^2 - n^2)$.

Section 9. — value of v, value of N, value of R.

Step 3 — note (Page 4) an arithmetical progression — Page 3. c tables, Section 6. The conception of the values of these areas forming an arithmetical progression, is the heart of this solution.

Section 6 gives the tables for Sections 5, 7, 8, and 9.

Section 9 and ½. Recognition of ideas of Pythagoras

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(Title) — A Guide Thru the Progressive Steps —

Step 4 — Chapter III — Adolescence

Section 10, Article I — The value of a division defined. It equals the last difference, $(a^2 - n^2)$. Arithmetical Progression discussed further.

Article 2 — The bottom row of differences are a descending arithmetical progression from $(a^2 - n^2)$ downward. The row of differences, above are an ascending arithmetical progression. Both of these form a complete arithmetical progression from d^2 thru $(a^2 - n^2)$. The ideas of Section 10, pages 5 and 5.b are the final considerations which make Integral Diagrams a reality.

Step 5 — The Integral Diagram —

Sections 11, 12, 13, and 14. Setting up Integral Diagrams. Illustration page 6. b. Top of Page 7 — an Integral Diagram defined. Section 15 — The smallest Integral Diagram possible would have $\frac{1}{3}$ of one division below n^2.

Note: The reason for Integral Diagrams is that they give us a means to deal with problems. in which a or N is not divisible by $(a - N)$ or, in other words, d. A second difference, (l.s.), is injected into the linear d series of differences. The diagram of such a series is called a complete Diagram.

(Title) — a Guide Thru the Progressive Steps —

The Integral Diagram give us a way to separate these new area values from the arithmetical series in d^2 values. This is did by subtracting the Integral Diagram (in terms of divisions) from the Complete Diagram

The complete separation of these two sets of values in the complete diagram remains thru the entire remainder of this solution

Step 6. a — (Preparatory) — The value of the sum of any groups of differences = The sum of the d s in the group times twice the "centerpoint" of the group. Sections 17, 18, 19, and 18. a.

See Rule I and Rule 2 — Page 8, Section 18.

Note: These considerations are important because they make possible the idea of the "Big Space". Section 20 — "Broken Series" — These values, caused by the insertion of the value, (l.s.), into the linear series; are inserted into the arithmetical series of differences, (surfaces), by means of the "Big Space". (Called "Broken" because of the insertion of a "l.s." value into the linear ~~series~~ "d" Series of differences.)

Step 6. b. — "The Big Space" — The "Big Space" is the area grouping caused by the insertion of the l.s. difference into the linear "d" series of differences. L.S. means "little space" when used to designate an area. Sections 22 to 32.

Page VIII

Laurence M. Rask
Union Iowa

(Title) — a Guide Thru the Progressive — Diagram — page 10.a.

→ Note: The "Large Space" and the "Little Space" can be placed in any relative position to each other, without effecting the values of the "d" differences of the arithmetical Progression, so long as they both are within the Big Space.

These Calculations collaborate my conception of the arithmetical Series.

~~Se~~ Section 33 — These Calculations will be needed later to calculate the "l. s. column."

Sections 34 and 35 — The Triangular Series.

Step 7. — Subtraction of the Integral Diagram from the Complete Diagram — The l.s. column.
Page 16, Article I.

Article 2 — Pages 17 and 18, Section 37.

The Procedure of Article 2 is very important. This is the calculation of the Integral Diagram inside the Complete Diagram. This operation permanently divides the Complete Diagram into two parts — The Integral Diagram and the (l.s. column). This division can be achieved even when N is unknown or is a surd.

Page 18.a is the advanced rearrangement. of Page 18.b.

Pages 18.c, 18.d, and 18.d.a are explanatory.

Page IX

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Union Iowa

(Title) — a Guide Thru the Progressive Steps — Page 18.e, Section 37½ — Determination of the exact place of the insertion of (l.s.c.) into the arithmetical series of d^2 differences. Page 18 and ½, Section 37 and ¾ — Restatement of the problem.

Note: Page 19, Section 38 — concerning Barriers.

Step 8 — Pages 20, 20.a, 20.b, and 20.c (all in Section 39) — a rearrangement of (l.s.c.) so as to discover its parts — as the G-Space, S, K, L, etc.

Step 9 — Pages 21, 21.a, and 21.b — all in Section 40. This is an effort to express (l.s.c.) in terms of G. Article III — This value is called (T.V.), True value.

Step 10 — Analysis of G-Space — Section 41, Pages 22, 22.a, 23, 23.a, and Page 24.

Note: at the bottom of Page 24 is the Rule: There are as many ±s in L, as there are Gs in (l.s.c). The finding of this value, (T.V.), is the key which unlocks the solution of all square roots. Section 42 — numerical proof of value in (l.s.c) Pages 25, 25.a, 25.b, 25.b.a, (abstractly) 25.c, 25.d, (verification) 25.e, 25.f.

Notice: There is no Section 43.

Section 44 — Two examples of the numerical proof of the Rule: "There are as many ± in L, as there are Gs in (l.s.c.)" — Example I — Pages 26, 26.a, and 26.b. Example 2 — Page 26.c.

Page X

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(Title) – a Guide Thru the Progressive Steps –

Step 11 – Section 46 – The expression of the (column of K's) as a (Sm. Inc. v.) G – Page 27.
Sec. 46 and $\frac{1}{2}$ – page $27\frac{1}{2}$ – Outline of remainder of Proof.

Note: The (Small Inc. value) L.S.C. at the (Sm. Inc. value of G) Rate, may be expressed in values of ±. This is a rectangular surface. This is the conception of (Small Inc. v.) L.S.C. on which Step 12 (The Excess of ± Theory), Step 13 (The Segregation Diagram), Step 14 – The Z Theory, and Step 15 (Intangibles), are based. The explanation of this group of ideas extends from page 27 to page 35, inclusive.

Step 12 – The Excess of ± Theory {is a comparision between the verticle and horizontal expansions of (Sm. Inc. value) G at the (Sm. Inc. value) Rate.}
Page 28, Section 47 – Two kinds of ± s.
Section 48 – The Excess of ± Theory – Pages 29 and 29.a
Section 48, Article III – a numerical calculation of the consequential Increase. – Page 30.a.
Page 30.f – Comparision between added Increase and consequential Increase.
Page 30.g – agreement with Excess of ± Theory.

Step 13 – The Segregationg Diagrams – Section 49 – Pages 31 and 31.a
Subtraction of $\frac{1}{y}$ of a G – Explained
Page 32, Section 50.

Page XI

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(Title) – A Guide Thru the Progressive Steps –

Note: The Segregating Diagrams are a series of Diagrams whose purpose is to separate {Remainder of (T.V. l.s.c.) at the (T.V. Rate)} from W and its derivatives.

We are now ready to calculate {Remainder of (Inc. v. l.s.c.) at the (Inc. value) Rate} in terms of Z.

Step 14 – The Z Theory is the expression of the Remainder of (Inc. v. l.s.c.) in terms of added Increase. The formula is $(Z + 2 + \frac{1}{2})$ Section 5.1, Page 32.

Step 15 – Intangibles – Pages 33 to 36.c, Section 52 to Section 53.

The formula $(Z + 2 + \frac{1}{2})$ can be reduced by subtracting 2.

Z = {Remainder of (T.V. l.s.c.) at the (T.V. Rate)}

$\frac{1}{2}$ = W. These are the first Intangible values

Method I – The developement of the rest of the series is explained in the text, Section 52, Pages 34 and 35. The series may be developed until the derivative of W becomes so small as to be ignored. Then calculate back to the {Remainder of (T.V. l.s.c.) at the (T.V.) Rate}, thus getting a value for Z.

Step 15 and $\frac{1}{2}$ – methods based on "Breaks".

Page 37, Section 53 – (continued) to Page 37.a – Section 53.

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(Tital) — a Guide Thru the Progressive Steps —

This is the finding of a "break" or vacant (or almost vacant) series of digits (zeros), in the numerical value of an Intangible, between the "Z" and "$\frac{1}{Z}$" values. as in (Z and $\frac{1}{Z}$) First Intangible; (Z^2 and $\frac{1}{Z^2}$) Second Intangible; (Z^4 and $\frac{1}{Z^4}$) Thrid Intangible, etc. etc.

Pages 37. ba to Page 37. d.

(These methods are interesting but they are not the best — Pages 38 to Page 40 — Section 54)

Steps 16 — method III. b. b. — Pages 41 and 42 — Section 54 continued. These Calculations are based on the first Intangible values. This is the method which will get accurate results the easy way.

This is the end of my Theory. The rest is only the completion of indicated Calculations.

Chapter VI — Page 43, Section 55.
Will This Theory Endure "The Trial by Fire"?
Extraction of the Square Root of a Surd.

Laurence M. Rash
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II – Index to the Inherent Ideas of This Theory.

Page 4 and ½ – In Recognition of Pythagoras.

Page 19 – Barriers,

Page 45, Section 57: – Barriers.
  A – Interdimensional Barriers.
  B – Intra dimensional Barriers.
    (I) – The abstract, qualitive, or theoretical.

Page 46, Section 58: – The Spiritual and the Natural.
    (II) – The quantitive, numerical, or known.
  C – Conversion of abstract value of (I) into numerial values of (II).

Early February, 1928

Part I

Chapter I – Birth

(The Origin)

– The Creative idea and beginning of the Problem –

Section 1. a – The "funny little table."

1 = 1
1 + 2 = 3
1 + 2 + 3 = 6
1 + 2 + 3 + 4 = 10
1 + 2 + 3 + 4 + 5 = 15
1 + 2 + 3 + 4 + 5 + 6 = 21
1 + 2 + 3 + 4 + 5 + 6 + 7 = 28
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45
etc. etc. etc. etc.

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Early Rebruary 1928

Section I .b

One evening I was seated in an old rocking chair in the kitcken of my father's home, which was also my home, as I was still living with my parents. My mind was concerned with no special problem, but drifting idly from one thought to another. Suddenly the "funny little table"on page I.a appeared on the stream of consciousness.

Iwas much interested and began to examine the table logically and critically. A little experimentation showed that "The sum of all the integers less then a give integer equals the product of the next lower integer times one-half of the given integer. "
Thus: The sum of all the integers less then nine equals 8 times four and one-half. The product is 36 .

The phrase "one-half of the given integer" made me wonder, "What would happen if the entire integer were used?"

The rule is :--
The square of an integer equals the product of the given integer times the next lower integer plus the given integer.

That is:-- Let inte. = integer

$$\text{inte}^2 = (\text{inte}) \times (\text{inte} - I) + \text{inte.}$$

$$\text{inte}^2 = (\text{inte} + I)\ (\text{inte} - I) + I$$

Let d = the difference between the given integer and the numbers used.

Then $\text{inte}^2 = (\text{inte} - D)\ (\text{inte} + d) + d^2$ Formula I

Experimentation shows that this formula is true for any value of d .

Rule I ; --The square of any integer equals the integer minus a difference times the integer plus that difference, plus the square of that difference.

Section 2

When I found formula I, it seemed to me that every rational number must have a rational square root. Itried to find a method of extracting the exact square root of any number. (After consulting high school and college algebras and my own experience; I do not think so now.)

I thought my formula was unique, unrelated, isolated, solitary because of the manner in which I developed it.

It is fortunate I had that delusion. Had I known the possibilities of such great difficulties, Imight not have tried.

Christopher Columbus had comparative delusions. (I) He believed the earth was round; but (2) the earth was much larger than he thought the distance to India much greater; and(3) the Americaan contiments lie across his path. Had he known, he might not have tried.

Section 3.

Note: After some time I discovered there was a relationship between Formula I and the binomial theorm formula for squaring.

Let $n = (a - D)$.

Binomial formula $(a - d)^2 = a^2 - 2ad + d^2 = n^2$

Formula I $(n + D)(n - d) + d^2 = n^2$

Then $A^2 - 2ad + d^2 = (n + d)(n - d) + d^2$.

Subtract d^2 from both sides of the equation.

$A^2 - 2ad = (n + d)(n - d)$.

As $(n + d) = a$,

$a(a - 2d) = a(n - D)$.

$(a - 2d) = (n - d)$.

Therefor substituting one of these values for the other will transform either one of these formusls into the other.

formula 5

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Section 6

Early March to April 3, 1929

Tables of Differences of Squares

Let $d = 3$

1 number	2		3 squares	4 differences of squares	5 common difference of differences
0 —	0^2	=	0	9	
3 —	3^2	=	9	27	18
6 —	6^2	=	36	45	18
9 —	9^2	=	81	63	18
12 —	12^2	=	144	81	18
15 —	15^2	=	225		

difference = d, an abstract quantity

0 —	0^2	=	0	d^2	$2d^2$
d —	d^2	=	d^2	$3d^2$	$2d^2$
$2d$ —	$(2d)^2$	=	$4d^2$	$5d^2$	$2d^2$
$3d$ —	$(3d)^2$	=	$9d^2$	$7d^2$	$2d^2$
$4d$ —	$(4d)^2$	=	$16d^2$	$9d^2$	
$5d$ —	$(5d)^2$	=	$25d^2$		

Let v = the ordinal ~~of the~~ number of the differences of squares (column 4)

Thus $9d^2$ = 5 th difference of squares.
$= 4(2d^2) + d^2$

Formula II :— The difference of any two squares $= (2v-1)d^2$.

Thus the value of any last difference can be found abstractly as $(2v-1)d^2$

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Union ,Iowa

Chapter II — Childhood

(The inexprienced wanderings and searchings for basic facts about the problem.)

Section 4 . Spring and Summer 1929

After working out Formula I, I kept trying to solve this problem by factoring. I got accurate results by trial and error methods, but not accurate enough to please me.

I tried to treat two variations of the same equation so as to ~~ha~~have equations suitable to proform eleminations. It failed.

Just befor Feb. 20, 1929

When two equations are but variations, theyboth express the same truths and fundamentals. No matter what new relations and ideas you may apply the old facts keep appearing.

Note:— This effort to find a solution by eliminations between two simultaneous equations is important because it was one of my efforts to cross the barrier between the first and second dimensionals.

Section 5 . Feb. 25, 1929

Rule; The difference between the squares of ~~the~~ two quantities equals the difference between the quantities times the sum of the quantities.

This is important because it introduces the conceiption $(a^2 - n^2)$.
See page 3.b It is over page.

Section 6 . Early March to April 3, 1929.

Rule: The differences between the differences of the squares of quantities having a fixed difference is twice the square of the fixed difference.

The tables on page 3 .c show that the value of $(a^2 - n^2) = (2v - I)\, d^2$ (Columns 3, 4, and 5)
This is an important formula .

Section 7. Spring and Summer 1929

In section 6 ,page 3 .c: — The quantities in column I, are to be regarded as one dimensional linear numbers. The quantities in the other columns are to be regarded as two dimensions surfaces.

Let v = the ordinal number of values in column 4 (differences of squares).

V also equals the linear number which is the top margin of the corresponding difference (column I). This enumeration does not count the zeros at the top of the column.

Thus $9d^2$ is the 5th difference of squares,
$(2v - I)\,d^2 = ((2x5) - 1)\,d^2 = 9d^2$ (column 4).
And 5d is the 5th linear difference (column 1).
In both cases $v = 5$.

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Section 5. Feb. 25, 1929

Let $n = 8$, $d = 1$ Then a would equal 9.

$(a^2 - n^2) = 17 = d\ (a + n) = 1\,(9+8)$

Let $n = 8$ $d = 2$ Then $a = 10$

$(a - n) = 2$; $(a^2 - n^2) = 36 = 2\,(10 + 8) = d\ (a + n)$.

Let $n = 8$ $d = 5$ Then $a = 13$

$$\left.\begin{array}{ll} (a-n) = 5\ ; \quad (a^2 - n^2) = (169 - 64) = & 105 \\ \qquad d\ (a+n) = 5(13+8) \quad = & 105 \end{array}\right\}\text{equal.}$$

Let $n = 7$; $d = 5$ Then $a = 12$

$$\left.\begin{array}{l} (a - n) = 5 \quad (a^2 - n^2) = (144 - 49) = 95 \\ \qquad d\,(a+n) = 5(12+7) = 5(19) = 95 \end{array}\right\}\text{equal.}$$

Let $n = 7$; $d = 2$ Then $a = 9$

$$\left.\begin{array}{l} (a^2 - n^2) = (81 - 49) = 32 \\ d\,(a + n) = 2\,(9+7) = 2\,(16) = 32 \end{array}\right\}\text{equal.}$$

Rule :— The difference between the squares of two quantities equals the difference between the quantities times the sum of the quantities.

This is important because it introduces the conception $(a^2 - n^2)$.

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Section 8 A ugust I3, I929

Table of section 6, page 3.c

Therefore $a^2 - n^2$, the last difference, equals the sum of all the common difference of differences before it (column 5) plus the first one-half difference.

Thus : 8I = 4 (I8) plus 9.

Section 9 . April I9, I929 and later

~~In column I~~ of section 6, Page 3.c
Table

In column I. Let (a− n) = d

If a= 5d then n = 4d D is common difference in this column.

The v of a = 5, meaning sum of 5 differences of d .

The v of n = 4, meaning sum of 4 differences of d.

N =
sum of 4differences ; A = sum of 5 differences.

When v of a =5 , a= 5d Then ~~v^2~~ $A^2 = v^2 d^2 = 25 d^2$.

Very important : − $A^2 = 25d^2$ = the sum of all the differences of squares, of which ($A^2_1 - N^2$) is the largest. (column 4)

Note:

The numerical values in column 4 , of the table at the top of page 3 .c, Section 6 form an arithmetical progression in which I8 is the common difference between consecutive terms.

The abstract values in D^2 units of column 4of the table in the middle of page 3.c, Section 6, form an arithmetical progression in which $2D^2$ is the common difference of consecutive terms.

Page 4 and I/2 — Inserted .
Section 9 and I/2 — Inserted .

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I n Recognition : —

I . Pythagor-as lived about 580 — 500 B.C . .

S2 I have known for a number of years that the early Pythagoreans had known of Gnomens.

3 . On February I9, 1965 Iwent to the public Library in Marshalltown to learn more about the matter.

4 /. A Gnomen is defined as "What is left of a parallelogram on removing a similar parallelogram containing any one of its corners. The parallelograms we are interested in are squares.

From Enclopaedia Britannica, Volume I8, year I962, Page 803 . Pythagorean Arithmetic ; I quote : — "The successive odd numbers after I were themselves called Gnomons because the addition of each to the sum of the preceding ones (beginning with I) makes a square number into the next larger square. "

6 . This is the very same series of differences as I presented on

Page 3 .c, Section 6 .

7 . I do not know if the Pythagoreans would have applied Gnomons to this problem the same as Sir Isaac Newton did in his Binomial Theorem ; or as the $(A^2 - N^2)$ Space as I have done.

Chapter III Adolescence

(In which the solution takes ~~definite~~ A definite pattern.)

Section IO. Nov. I2, 1929

Diagram, Page 5.b , Article I

Let a^2 or $v^2d^2 = 8I$. And $(2v - I)d^2 = 2$

As $(2v - I)d^2 = (a^2 - N^2) = 2$; $N^2 = 79$.
Then A^2 or $v^2d^2 = 40.5\,(2v - I)d^2$.
The term $40.5\,(2v - I)d^2$ can be expressed graphically as rectangle tsz y
See diagram page5.b ,section IO, article I .

The large rectangle tsz y,one side of which is $(2v - I)d$;and the other side, 40.5 may be conceived as made up of 40.5small rectangles whose unit dimensions are $(2v - I)d$ And 1. It must be understood that the numbers I and 40.5 are the coefficients of one linear d. These small rectangles equal to $(2v-I)d^2$ or 2 are called Divisions .

Article 2. By means of calculations which I am not ready to explain untill the next chapter, I find that $(a^2 - N^2) = I59\,d^2$. Going below N^2, the first difference is $(n^2 - c^2) = 157\,d^2$, the next lower difference is $(c^2 - c'^2)$ which equals $I55\,d^2$, the next lower difference equals $I53\,d^2$. This is a descending arithmetical progression of which each succeeding term is $2d^2$ less than the one before. This descending arithmetical progression continues until the right handend of the diagram is reached. This is the bottom row of series of differences. The top row of series of differencesbegins with the first, above zero and extends to the right hand side of the diagram. This ascending seires of series differences has its first term, d^2, placed justabove the difference $(N^2 - c^2)$ and exgents to the right hand side of the diagram. It will be seen that the sums $(n^2 - c^2)$ plus I d^2; $(c^2 - c'^2)$ plus $3d^2$; $(c'^2 - c''^2)$ plus $5d^2$; & & & & are equal. Thus there are 39.5 equal sums. Each space contains two terms whose sum is equal to the sum of the terms of any other space. Irefer to these parts of divisions below the row of d^2s, which contain sums as spaces. e sums ,

I have changed my mind about the calculations which I was not ready to explain. The right hand Division is only one half a division therefor is equal to only one half a sum $= 79\,d^2$. These two one half spaces also equal one difference $= 79\,d^2$. This is one way to calculate this difference. Also this is the 40th difference. $(2v - I)d^2 = ((2x40) - I)d^2 = 79\,d^2$. The two calculations a-gree.

Therefor I have established a arithmetical progression from d^2 thru to $(a^2 - n^2)$.

Page 5. f

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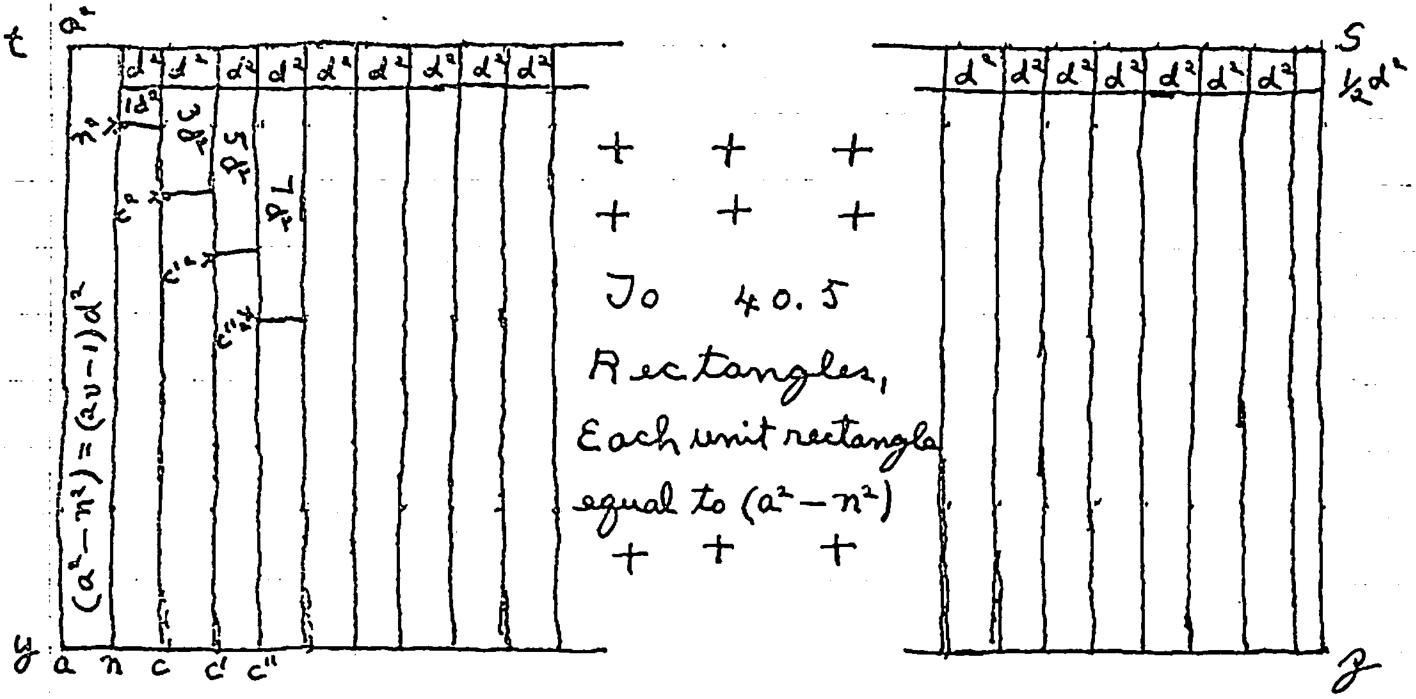
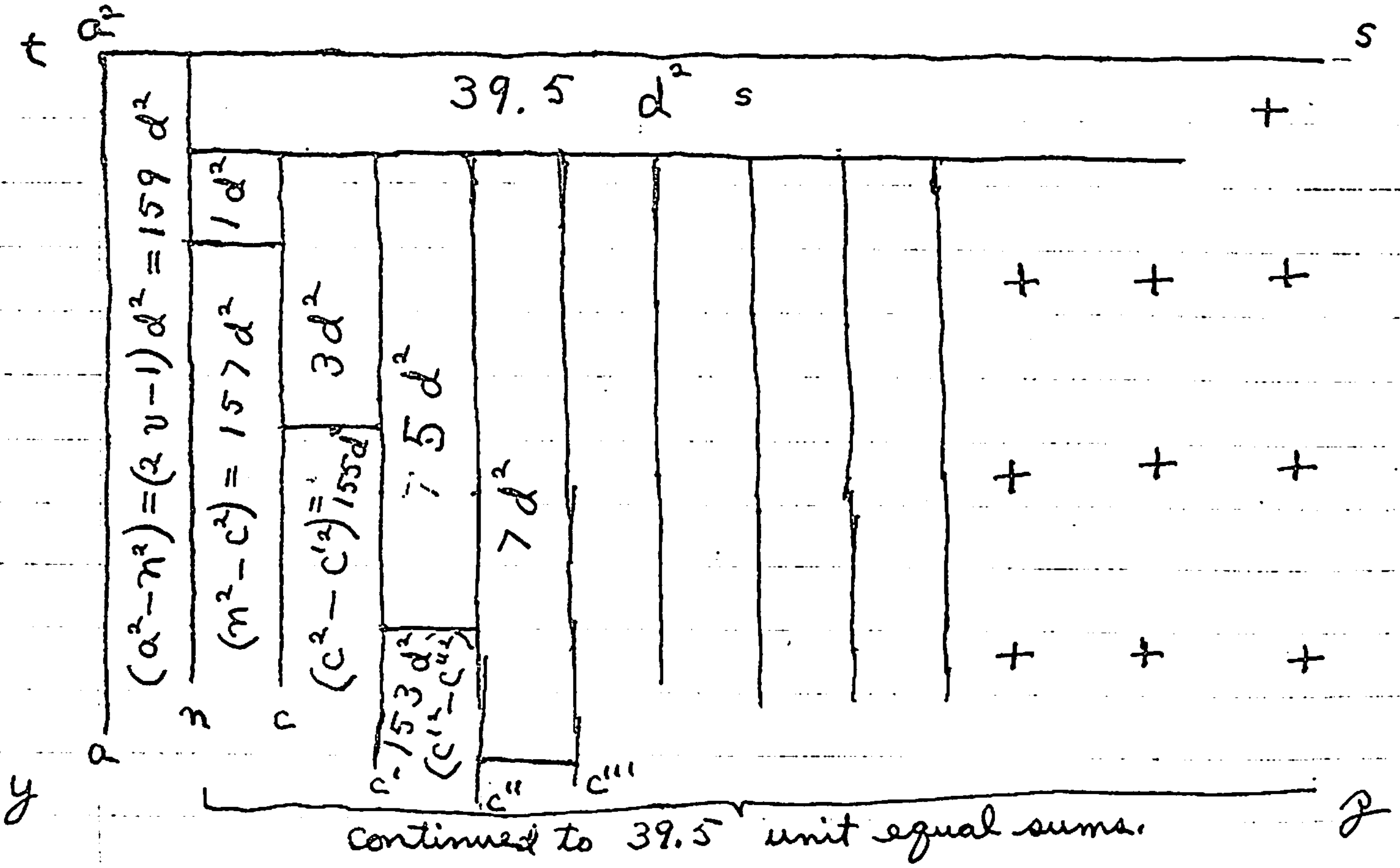
Union Iowa
Nov. 12, 1929

Section 10

Article 1



Article 2



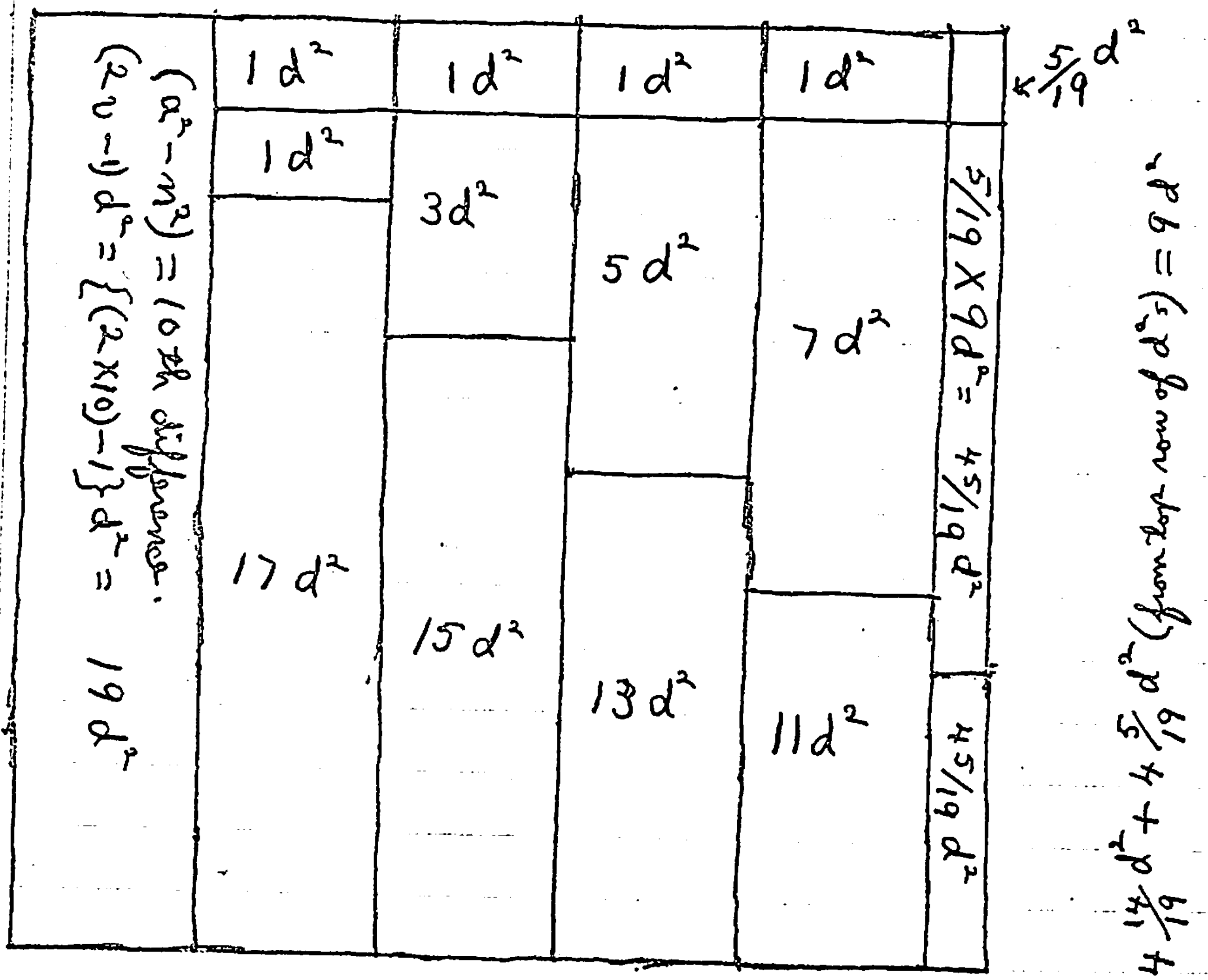
Laurence M. Rash
Union Iowa

Dec. 2, 1929

Section. 14. Integral Diagrams

Extract $\sqrt[2]{81}$ Let $a = 10$; $a^2 = 100$;
$n^2 = 81$; $(a^2 - n^2) = 19$

n is to be treated as an unknown value.
$81 \div 19 = 4$ and $5/19$ divisions below n^2.



It is easy to see that this diagram is not drawn to scale vertically. The $5/19$ division to the right is the 5th difference. $(2v-1)d^2 = \{(2 \times 5)-1\}d^2 = 9d^2$. $\frac{5}{19} \times 9d^2 = 4\frac{5}{19}d^2$. $4\frac{5}{19}d^2 \times 2 = \frac{90}{19}d^2 = 4\frac{14}{19}d^2$. To right margin. ↑

Section II.

The reason for the row of d^2 s across the top of the diagram is :— The difference (n^2 — c^2) is $2d^2$ less than ($a^2 - n^2$). The difference d^2 is added to complete the space and the sum. Therefor Id^2 is left in each division of the diagram.

Section I2.

This diagram is incomplete but it gives the basic ideas of my method of extracting square root. :—

N^2 is the number whose square root is sought.

whose
Nis the required square root. N is the required square root.

A^2 is any~~conv~~ convenient square, a little larger than N^2 ,whose square root, A, is known.
When once ~~a~~ A^2 is seclectedthe values A , (a— n), d , the arithmetical progression in D^2 & is fixed for the entire calculation.

Section I3

It will be noted that the row of D^2 is not included in the series of the arithmetical progression. That problem will be adequately solved later. There are two other important omissions probably not noticed by the reader.

Section I4. Integral Diagrams.

As an illustration ,let us take $N^2 = 8I$, ~~treat~~ but treat N as its unknown square root. Take A^2 as I00, its square root, I0 is a known value.

$(A^2 - N^2) = I9$. $8I \div I9 = 4$ and 5/I9. There are 4 and5/I9 divisions below N^2. The 5th difference is calculated from The fractional 5/I9 space to the far right. 2(5/I9) = I0/I9. ~~2~~(2v— I) D^2 =

((2x5)— I)D^2 = 9D^2. = I7I/I9 D^2. (I0/I9)(9D^2) = 90/I9 D^2 from space.
from

The 4 and 5/I9 D^2 from the rowof D^2 = 8I/I9 D^2

8I/I9 D^2 plus 90/I9 d^2 fromthe space = I7I/I9 D^2 .

The two calculatioms Agree. This is a complete difference.

Figuring the diagram abstractly;—

AS the 5th difference is complete ,$(a^2 - N^2) = I9\ D^2$.

AS there are 9 differences below N ; the $\sqrt{d^2}$ Of N^2 = 8I D^2

8I D^2 ÷ I9 D^2 = 4 and 5/I9 Divisions ,found abstractly.

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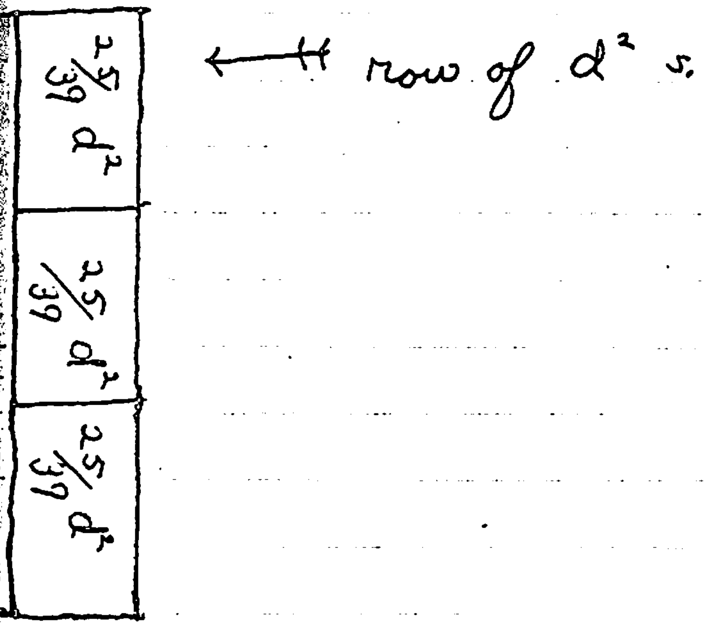
15. July 4, 1930.

A trial diagram.

$= 64$; $n^2 = 25$; Then $(a^2 - n^2) = 39$.

$39 = {}^{25}/_{39}$. There is a ${}^{25}/_{39}$ division below n^2.



Section I4 —Continued.

The reason these diagrams are called Integral Diagrams is that d is an exact divisor of both A and N . There are no fractional differences. The entire arithmetical progression is expressed in D^2 values ,from the initial D^2 thru to ($a^2 - n^2$). All values are integral.

Section I5 . ~~Atrial~~ July 4, 1930.

A trial diagram:—
Let $A^2 = 64$, $N^2 = 25$. Then $(A^2 - N^2) = 39$.

~~25~~ ~~25~~ 25 ÷ 39 = 25/39. There are 25/39 divisions below N^2.
The ~~frist~~ first difference is Id^2. Therefor 25/39of d^2 = 25/39 d^2.

I4/39 d^2 is taking from the top row of d^2 s, to complete the first difference.
, leaving II/39 d^2 inthe top row of d^2.
The second difference would be $3d^2$. 25/39 of $3d^2$ = 75/39 d^2.

75/39 d^2 + the II/39 d^2 fromthe row of d^2 s = 86/39 d^2 equals

2 and 8/39 d^2. Ihave failed to complete the second difference of 3 d^2. Therefor my theory was wrong. There is only one of the d^2 series of differences below n^2 .
That this diagram is not a integral diagram is easily seen , for A = 8 and N = 5 . (A — N) = 3 . Three is not an exect divisor of either 8 or 5.

The correct calculation:

There~~are two~~ is a sum of two 25/39 spaces in the first division.
2x 25/39 = 50/39 of a space. 50/39 of Id^2 = 50/39 d^2.
50/39 d^2 — Id^2 = II/39d^2 .
II/39 d^2 + the 25/39 d^2 ,from the row of d^2, = 36/39 d^2 left in first division. This problem must be left for solution later.
The difference $(a^2 - n^2)$ = 3 d^2.

Note: The value d must be determined by actual construction of the differences of the right hand division. the value v depends on the numberof differences in the right hand division.

~~SectiThe Arithmetical~~
Section I6 .

The arithmetixal progression, ~~A^2 to~~ D^2 to $(A^2 - N^2)$ is a partial solution of the diagram just above. The deficiency is confined to 36/39 d^2. This is quite a deficiency,but still it is confined.

PART II

Chapter IV

Young Adult – Hood

Page 7 and I/2 Inserted later Laurence M. Rash

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Part II

In this part I shall use known values of N, D, D^2 , (l.s.), ch, etc. But treat them as unknown values as much as possible to work out formulae . But at the same time these known values will be available to prove my assertions. Inthe next three chapters I shall try to work out formulae , laws, relations, rules, and methods .

, s .

In Part III I shall attempt to extract the square root of a surd to a very close approximate, &I shall have to depend entirely on the methods, ~~on the med-hods~~ and principles I have worked out.

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Chapter I V Young Adult - Hood.

(in which all the values of a complete diagram are found a bstractly.)

SEction I7 . June 5 , I93I.

Rule:- At any given " center point " the " d " varys directly as the number of d^2 s in any given difference of squares. The thought is that succeeding differences of squares may be conbined in a group and a large "d " (the number of differences in the group) may be considered the " d"of the group. This large "d" is ,of course, alinear first dimensional value.

The "centerpoint" is the middle of the large "d "on the frist dimensional scale.

Thus: (illustration Page 8.b) cne group iS $3d^2 + 5d^2 + 7d^2 = I5d^2$. The "centerpoint is within the ~~differec~~ difference $5d^2$ The "centerpoint"of thelarge "d " is at 2and I/2d ~~on~~ on the frist dimensional ,linear scale. The large " d " equals 3d.

For the f~~ri~~irst group the proportion is $I5d^2$; $5d^2$ = 3d : Id.

The proportion for the second group is 5 $6d^2$: $I4d^2$ = 4d : Id.
$56d^2$ is the sum of the differences in the group.

For $I4d^2$:- The centerpoint ofthis group is at the top of the 7th difference. Therefor the "centerpoint "is 7d~~and I/2~~d. (2 v - I) = ((2x 7 and I/2) - I) = I4. The large "d " = 4d.

Note: T nis is a uni t difference so place that it overlaps on boththe 7th and 8thdifferences . v= 7 and I/2d.

For the $I4d^2$difference :- The linear d extends from (I3/2)d to (I 5/2)d

If the centerpoint is at the middle of the 8th difference :
A one d width gives I5 d^2.
A five d width gives $IId^2 + 13d^2 + I5d^2 + I7d^2 + I9d^2 = 75d^2$.
5 x $I5d^2 = 75d^2$.

Section I8. March 7th. and 8th. I93 2.

Rule I : The number of units in a difference varys directly as its position in its series. By position, I mean the position of its centerpoint.

Rule 2 : Any term in the same series equals 2d x its centerpoint.

To illustrate : The first difference is d^2. Its centerpoint = I/2 d.
2d xI/2d = Id^2. The 3rd difference = $5d^2$. 2d x2 and I/2 d = $5d^2$.
The 7th difference = $I3d^2$. 2d x 6and I/2d = $I3d^2$.

Section I9. Agreement between the two formula for calculating the differences of squares.

4 th. difference = $7d^2$ v= 4 . $((2v - I)d^2) = 7d^2$.
Centerpoint = 3 and I/2 d. 2d x 3and I/2 =d $= 7d^2$. $7d^2$.

9-th. difference = $17d^2$ v = 9. $(2v-1)d^2 = 17d^2$
Centerpoint = $3\frac{1}{2}$ d. 2d x $8\frac{1}{2}$d = $17d^2$

Page 8. b

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Chapter IV Young Adult-Hood.

(In which all the values of a complete diagram are found abstractly.)

Illustration for sections 17, 18, and 19.

Ordinal linear differences
1st dimensional values } first row values

"d" = 3d
centerpoint 2 1/2 d

"d" = 4d
centerpoint = top of the 7th difference = 7d.

1st. 2nd. 3rd. 4th. 5th. 6th. 7th. 8th. 9th. 10th.

$$d^2 \quad \underbrace{3d^2 \quad 5d^2 \quad 7d^2}_{\text{group 1}} \quad 9d^2 \quad \underbrace{\underbrace{11d^2 \quad 13d^2 \quad 15d^2 \quad 17d^2}_{\text{group 2}} \quad 19d^2}_{\text{group 3}}$$

Differences of succeeding squares
$\underbrace{\text{2nd dimensional values}}_{\text{2nd row of values}}$

Linear difference 1st. 2nd. 3rd. 4th. 5th. 6th. 7th.
centerpoints $\frac{1}{2}d$ $1\frac{1}{2}d$ $2\frac{1}{2}d$ $3\frac{1}{2}d$ $4\frac{1}{2}d$ $5\frac{1}{2}d$ $6\frac{1}{2}d$

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Sextion I8.a An addition to Section I8

If the centerpoint is expressed in numerical values instead of d values Rule 2 would read " Any term in the series equals $(\uparrow 2 D^2)$ x its centerpoint ."

Section 2o.

Under Section I4 , we discussed what I call Integral Dia grams. In this example there are 9 differences below N^2 . $(A^2 — N^2) = I9\ d^2$. This is an arithmetical progression from Id^2 thru $(a^2 - n^2)$. All d^2 values are a part of the arithmetical progression. This is tantamount to a solution of all unknown values of the diagram.

"Broken Series" Under Section I3 — " It will be noted that the row of d^2 s is not included in the series of the arithmetical progression". Somewhere between Id^2 and $(a^2 - n^2)$ these values must be inserted in the series. Therefor I call it a "broken series". The task ofthis present chapter is to relate these values to the values of the former arithmetical progression series, and insert them in the series.

Section 2I .

August 27, I93I.

I tried to solve placement of the row of d^2 s by studying the ~~por~~ proportion between two parts of an arithmetical progression which had been divided. Having not succeeded in this, by October 9, I93I — I combined with this the idea of placing the differences in larger groups, or else Ealwangrshdydtoagfractponn..~~thefidhe Sorfindfng~~ Byafebrurary 26, 192632I was making the d' a fractional of the former d.

By February 26, I932 I was ready to give up on the idea of finding what fractional of the original d the d' of the row of d^2 s was. Consider: —

(I) It is easy to establish this relationship when N is known; and N and D are commensurable in fairly large units.

(2) It is more difficult to establish this relationship when N is known; but N and D are incommensurable except in very small units.

(3) It is impossible to establish this reationship DIRECTLY, when N is an unknown value; and it is not known whether N and D are commensurable at all or not.

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Sections 22 and 23 March 5, 1932

The "Big Space" idea.

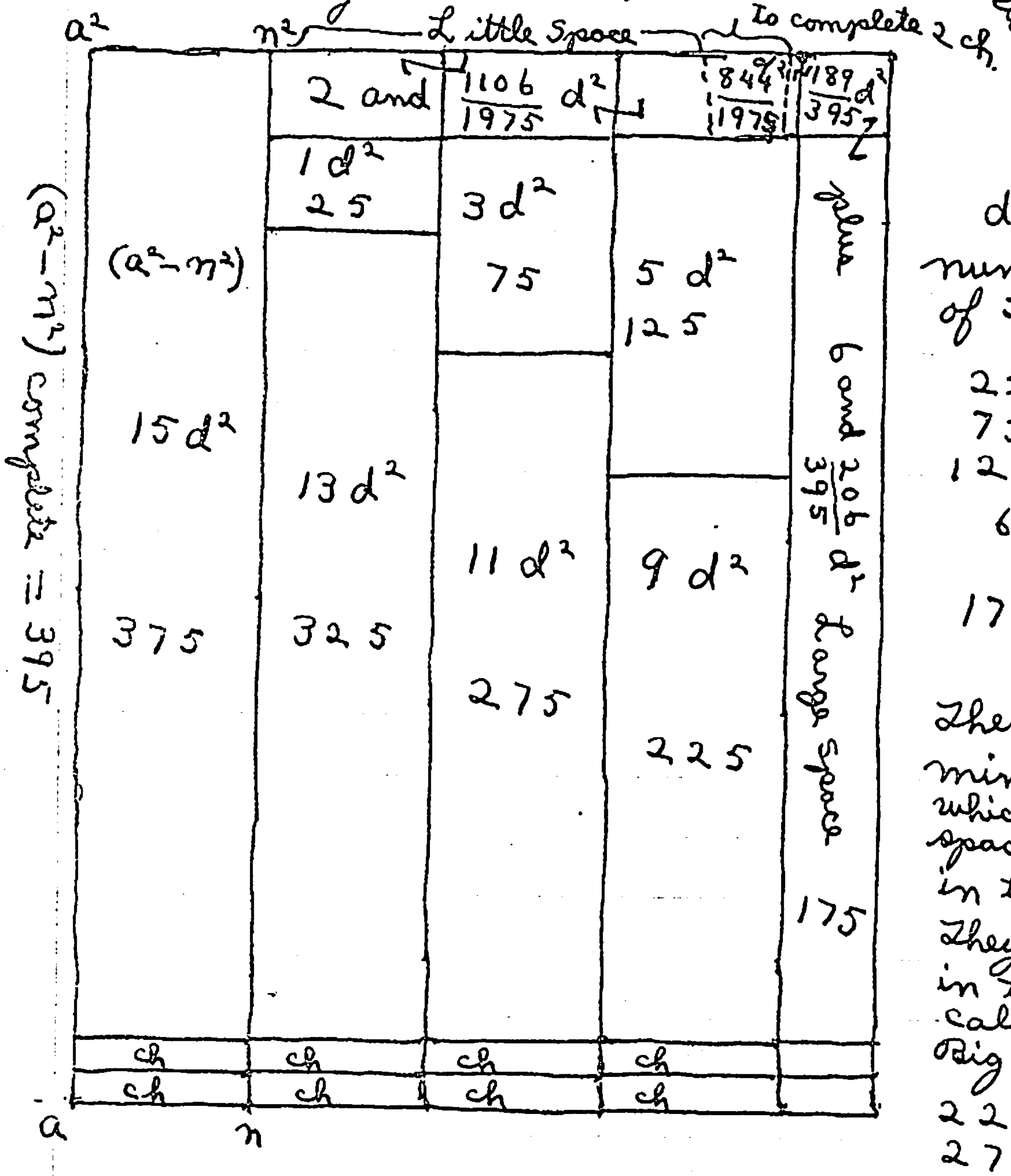
Let $a = 42$, then $a^2 = 1764$

Let $n = 37$, then $n^2 = 1369$

subtracting $(a^2 - n^2) = 395$

These are divisions of n^2: 3 $\frac{184}{395}$ below

$$395 \overline{)1369} (3$$
$$\frac{1185}{184}$$



$d^2 = 25$

numerical values of spaces

25	
75	
125	
64	Little Space
175	Large Space

These two are minimal values which occur when the space is placed first in the Big Space. They can not occur in the same calculation of the Big Space.

225 + 2ch
275 + 2ch
325 + 2ch
375 + 2ch

$$ch = (d \times l.s.) = (5 \times 2) = 10$$

Section 22. March 2, I932.

Let $A = 42$, Then $A^2 = I764$.
And $N = 37$, Then $N^2 = I369$.
Then $(A^2 - N^2) = 395$. $d = 5$

$I369 \div 395 = 3$ and I84 /395. Therefor there are 3 And I 84 /395 divisions below N^2.

Section 23.

This diagram is calculated by the use of the numerical values for $(A^2 - N^2)$ and N^2. Notice that the abstract D^2 values are not used. This is a complete diagram.

Section 24.

Itis proven that there is only one full d difference in the fractional division to the right in D^2 values + THE D^2 value in the row of D^2 s.

Section 25.

By completing the full d difference of the fractional division to the right, we have an arithmetical progression in D^2 values from zero thru The $(A^2 - N^2)$ difference.

Section 26.

The space in the fractional division to the right must have value from the row of D^2 sadded to complete it into a full d difference.

Section 27.

The value left in the row of D^2 s has a small d value of its own, called " Little Space " or "ls".

Section 28.

The top row of full d differences still form an arithmetical progression. One point to insert the "Little Space" is just befor the full d difference in the fractional division to the right + (the added value from the row of D^2 s) —to complete it of course.

Section 29. "The Big Space idea " March 5, I932.

Article I. The full d difference from the fractional division to the right is called the "Large Space ". The Large Space + the Little Space = the Big Space. The Big Space has a " d " value of its own equal to (d + l.s.).

Article 2. The Large Space and the Little Space can be placed in any position relative to each other within the Big Space, provided the proper value is assigned to each.

Article 3. Their relative positions within the Big Space may be changed if the proper value is interchanged from one to the other.

Article 4. This value equals (d x l.s.). It is called "changed by shifting " and is designated by "ch " .

Page 10. b

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Section $\overset{22}{:}$ —

$(a - n) = (42 - 37) = 5 = d$

$42 = a$ $\qquad$ $37 = n$

$$\begin{array}{r} 42 \\ \underline{42} \\ 84 \\ \underline{168} \\ 1764 \end{array} = a^2 \qquad \begin{array}{r} 37 \\ \underline{37} \\ 259 \\ \underline{111} \\ 1369 \end{array} = n^2 \qquad \begin{array}{r} 1764 \\ \underline{1369} \\ 395 \end{array} = (a^2 - n^2)$$

$$395 \overline{)1369} (3 \qquad \begin{array}{r} 1369 \\ \underline{1185} \\ 184 \end{array}$$

There are 3 and $\frac{184}{395}$ divisions below n^2

Section 24: There are two $(\frac{184}{395})$ fractions of a space, arranged one above the other in the fractional division. $2 \times (\frac{184}{395}) = \frac{368}{395}$ of the space. This space would be the 4th difference and equals $7d^2$.

$$\begin{array}{r} 368 \\ \underline{7} \\ 2576 \end{array} \qquad 395\overline{)2576}(6 \quad \begin{array}{r} 2576 \\ \underline{2370} \\ 206 \end{array} \qquad 6 \text{ and } \frac{206}{395} d^2 \qquad \begin{array}{r} 395 \\ \underline{206} \\ 189 \end{array}$$

$\frac{189}{395} d^2$ must be taken from the row of d^2 s.

There is not nearly enough d^2 value left in the row of d^2 s to form the second space or difference.

Section 25 ⇒ Therefor $(a^2 - n^2)$ is the 8th difference and equals $15 d^2$ in the integral series.

Section 26: $\frac{189}{395} d^2$ is added.

Section 29: "The Big Space Idea" March 5, 1932

Article 1

The difference of Large Space = d
The " " " of Little Space = l.s.
The "d" difference of Big Space = (d + l.s.)

In this example: d = 5; l.s. = 2
(d + l.s.) = 7

The Big Space extends from 3d to (4d + l.s.)
3d = 15 (4d + l.s.) = 22

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \end{array} = (15)^2 \qquad \begin{array}{r} 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline 484 \end{array} = (22)^2 \qquad \begin{array}{r} 484 \\ -225 \\ \hline 259 \end{array}$$

259 is the numerical value of the Big Space.

Articles 2 and 3

It makes no difference what positions the Large Space and the Little Space are placed, relative to each other, WITHIN the Big Space, their sum is always the same.

In this example this sum = 259.

Article 4

This value equals (d X l.s.) = ch
In this example (5 X 2) = 10 = ch

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Sections 30 and 3 I :

Separation of Large and Little Spaces.

It was necessary to separate the Large Space from the Little Space so as to analyze the row of d^2 s into I89/ 3 95 d^2 to complete the Large Sparge, and 844/I975 d^2 to complete 2ch.

I need this analysis to solve the problem of how differences in the diagram were to be calculated.

Please note that it would be impossible to make these separations in either d^2 units or in numerical units, if N were an unknown quantity. If N were unknown then the value of d, d^2, (l.s.), (l.s. $)^2$, ch, etc. would be unknown.

Having evolved the theory, I was able to go on to the next step. This knowedge helped me to understand the mechanics of the Big Space.

Page 11.a

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Section 30: Separation of Large and Little Spac

The Large Space is the 4 th. difference, $= 7d^2$,
It occupies both the upper and lower halves o
the $\frac{184}{395}$ division. Then $2 \times \frac{184}{395}$ space $= \frac{368}{395}$ sp

$$\frac{368}{395} \times 7d^2 = 6\ \frac{206}{395}d^2$$

$$\begin{array}{r} 368 \\ 7 \\ \hline 2576 \end{array} \qquad 395\overline{)2576}(6 \quad \begin{array}{r} 2370 \\ \hline 206 \end{array}$$

$$\begin{array}{r} 395 \\ -206 \\ \hline 189 \end{array}$$

Then $\frac{189}{395}d^2$ must be subtracted from row of d

$3\ \frac{184}{395}d^2 - \frac{189}{395}d^2 = 2\ \frac{390}{395}d^2$ left in the row of d^2 s

Now going from d^2 s to numerical units.
as $d^2 = 25$

$$\begin{array}{r} 390 \\ 25 \\ \hline 1950 \\ 780 \\ \hline 9750 \end{array} \qquad 395\overline{)9750}(24 \quad \begin{array}{r} 790 \\ \hline 1850 \\ 1580 \\ \hline 270 \end{array}$$

$\frac{390}{395}d^2 = 24$ and $\frac{270}{395}$ numerical units.

numerical value left in row of d^2 s.

$\frac{390}{395}d^2 = 24$ and $\frac{2}{39}$
$2d^2 = 50$
Total 74 and $\frac{2}{39}$

Figuring ch

The $\frac{184}{395}$ division has $2 \times \frac{184}{395}$ ch., equals $\frac{368}{395}$ ch. under it. There must be 2 ch.

$(1\text{ ch.} + \frac{395}{395}\text{ ch.}) - \frac{368}{395}\text{ ch.} = 1\text{ ch.} + \frac{27}{395}\text{ ch.}$ to b
taken from the row of d^2 s.

$(1\text{ ch.} + \frac{27}{395}\text{ ch.}) \times 10 = 10 + \frac{270}{395}$ numerical units

— From Page 11. a

Figuring the Little Space in position above the Large Space.

numerical value left in row of d^2

$$= 74\tfrac{270}{395} \text{ units}$$

Subtracting numerical value of $(1ch + \frac{27}{395})ch = 10\frac{270}{395}$ units

$$64$$

Little Space equals the remainder, 64 numerical units.

As the Large Space follows the Little Space ~~the~~ in the series, the Large Space is increased 2 ch.

Section: 31 Refiguring Section 30 to obtain Little Space by means of abstract d^2 values.

There are $3\frac{184}{395}$ d^2 s in row of d^2 s.

(1) Then $\frac{189}{395}$ d^2 is subtracted to complete Large Space, leaving $2\frac{390}{395}$ d^2 in row of d^2 s.

(2) Then $(1ch + \frac{27}{395})$ is taken to complete 2 ch.

$(1ch + \frac{27}{395}ch) = \frac{422}{395}ch$ $\qquad 1ch = \frac{2}{5}d^2.$

$\frac{2}{5} \times \frac{422}{395}ch = \frac{844}{1975}d^2$ = value taken to complete 2 ch.

(3) $2\frac{390}{395}d^2 = 2\frac{1950}{1975}d^2$

$2\frac{1950}{1975}d^2 - \frac{844}{1975}d^2 = 2\frac{1106}{1975}d^2$ = Little Space

(4) Check up $\qquad D^2 = 25$ numerical units.

$\cancel{25} \times \frac{1106}{\underset{79}{\cancel{1975}}} = \frac{1106}{79} = 14.$ $\qquad 2d^2 = 50$

$\frac{1106}{79} = 14$

64 = Little Space.

Section 32:

I. As the "Big Space" is one of the important "break thoughs" of my method, I wish to establish the principle so soundly that it will not be questioned. I shall try to do this by proving the values of the Little Space and Large Space in thre^{3}sets of positions in the Big Space. The calculations will be for the diagram on Page IO.a, Sections 32. 22 and 23.
Let a = 42, then a^2 = I764
Let n = 37, then n^2 = I369.
Subtracting, $(a^2 - n^2)$ = 395.

2. I have prOoven the relations between the Large Space and the Little Space exhaustively, for four pages. Ithink that all critics will have to admit that my proof is incontrovertible for this diagram alone. I think I have found the true principle for all convenient diagrams.

3. I intend to try operations with two other diagrams, one of which will be the attempted extraction of the square root of a surd. If I attain a high degree of accuracy, and in addition have a method which makes the possibilities of error approach infinitesmals, then my theory of the Big Space is proven.

4. On page 3. c Section 6, we have the tables for differences of squares in d^2s, column 4. For instance, by adding Id to 2 d, (column I), the difference is increased from 3 d^2 to 5 d^2. The common differences of differences is 2 d^2 = 2 (d x d), column 5. Now if instead of adding I d to 2 d; we add (l.s.) to 2 d, the common difference of differences becomes 2(l.s. X d) = 2 ch. This is but a corollary.

5. Altho it took me four pages to explain it, the mechanics of the " Big Space" are quite simple. AS I explained under 3, ; ~~If the large space was~~ (That was Section 29. Article 3.3), page I0. If the large space was befor the Little Space, then 2 ch would be added to Little Space. If the Little Space was Befor the Large Space, then 2 ch would be added to the Large Space. If they are both at the centerpoint of the Big Space, they each get I ch.

6. When I say that the Little Space and the Large Space are at the centerpoint of the Big Space, it means that the three centerpoint coincide. In Section 32: set two, Page I2.b —The Big Space is "built up" thus; (I/2 Large Space) — Little Space — last I/2 Large Space. Here the centerpoint of the Large Space is at the centerpoint of the Big Space, even tho this is in the middle of the Little Space, because the parts average of the Large Space average at this point.

7. It will be noticed that all the differences after the Big Space have 2 ch added to them.

values: —

$d = 5 \qquad d^2 = 25$

$l.s. = 2 \qquad (l.s.)^2 = 4$

$ch = (d \times l.s.) = (5 \times 2) = 10.$

Section 32:

1\. Set one: Little Space befor Large Space.

Little Space extends from 3d to (3d + l.s.)

$3d = (3 \times 5) = 15 \qquad (3d + l.s.) = (15 + 2) = 17.$

$(15)^2 = 225, \qquad (17)^2 = 289$

$$\begin{array}{r} 289 \\ -225 \\ \hline 64 \end{array}$$

Little Space = 64 ← They agree

Little Space centerpoint = $(3d + \frac{1}{2} l.s.)$

$(2\,l.s.)(3d + \frac{1}{2} l.s.) = 6\,ch + (l.s.)^2.$

$= 60 + 4 = 64.$

also agrees with the value found under Section 30, Page 11.b.

This is the minimal value for Little Space which occurs in this Big Space.

Large Space follows Little Space.

Large Space extends from (3d + l.s.) to (4d + l.s.)

$(3d + l.s.) = 17$

$(4d + l.s.) = 22$

$$\begin{array}{r} 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline 484 \end{array} \qquad \begin{array}{r} 17 \\ 17 \\ \hline 119 \\ 17 \\ \hline 289 \end{array} \qquad \begin{array}{r} 484 \\ -289 \\ \hline 195 \end{array}$$

value large Space

Centerpoint of Large Space = $(3d + l.s. + \frac{1}{2}d) = (3\frac{1}{2}d + l.s.)$

$2d(3\frac{1}{2}d + l.s.) = (7d^2 + 2\,ch)$

$= (7 \times 25) + 2(5 \times 2) = 175 + 20 = 195$

The two calculations for Large Space agree. This is the maximun value for Large Space in this Big Space. The sum of 195 + 64 equals 259 = Big Space. This agrees with the numerical value for Big Space Section 29, Page 10.c.

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Page 12. b
Section 32 – cont.:

2 Set two: Both Little Space and Large Space at the centerpoint of the Big Space.
The Big Space is "built up" thus: (½ Large Space, – Little Space – last ½ Large Space.

~~Frist~~ First ½ Large Space extends from 3d to 3½ d.
$(3d)^2 = 9d^2$, $3\frac{1}{2}d = \frac{7}{2}d$, $(\frac{7}{2}d)^2 = \frac{49}{4}d^2$.

$9d^2 = (9 \times 25)$
$= 225$

$$\begin{array}{r} 49 \\ 25 \\ \hline 245 \\ 98 \\ \hline 1225 \end{array}$$

$$4\overline{)1225}(306 \qquad \begin{array}{r} 12 \\ \hline 25 \end{array}$$

306¼ equals $\left(\frac{49}{4}\right) d^2$ numericall

$$\begin{array}{r} 306\frac{1}{4} \\ -225 \\ \hline 81\frac{1}{4} \end{array}$$

81 ¼ = numerical value of first ½ Large Space.

Second ½ Large Space
It extends from $(3d + \frac{1}{2}d + l.s.) = (3\frac{1}{2}d + l.s.)$ to $(3\frac{1}{2}d + l.s + \frac{1}{2}d) = (4d + l.s.) = (20 + 2) = 22$
$(3\frac{1}{2}d + l.s.) = (17\frac{1}{2} + 2) = 19\frac{1}{2} = \frac{39}{2}$

$$\begin{array}{r} 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline 484 \end{array} \qquad \begin{array}{r} 39 \\ 39 \\ \hline 351 \\ 117 \\ \hline 1521 \end{array} \qquad 4\overline{)1521}(380 \;\; \begin{array}{r} 12 \\ \hline 32 \\ 32 \\ \hline 1 \end{array} \qquad \begin{array}{r} 484 \\ -380\frac{1}{4} \\ \hline 103\frac{3}{4} \end{array}$$

Last ½ Large Space = 103 ¾ numerically.

$$\begin{array}{r} 103\frac{3}{4} \\ +\ 81\frac{1}{4} \\ \hline 185 \end{array}$$

Numerical value of Large space at this centerpoint = 185.

Centerpoint of Big Space = $(3\frac{1}{2}d + \frac{1}{2} l.s.)$
$2d(3\frac{1}{2}d + \frac{1}{2} l.s.) = 7d^2 + 1ch.$ $\quad 7d^2 = 175.$
$= 175 + 10 = 185.$

The two calculations agree.

Section 32 – cont.:
Set two – cont.

Little Space extends from $(3\frac{1}{2}d)$ to $(3\frac{1}{2}d + l.s.)$.
$(3\frac{1}{2}d)^2 = 306\frac{1}{4}$, $(3\frac{1}{2} + l.s.)^2 = 380\frac{1}{4}$
The values of these squares were calculated while calculating the Large Space of this set.

$$\begin{array}{r} 380\frac{1}{4} \\ -\ 306\frac{1}{4} \\ \hline 74 \end{array}$$

74 = numerical value at the Big Space centerpoint.

Centerpoint of Big Space $= (3\frac{1}{2}d + \frac{1}{2}l.s.)$
$2\,l.s.(3\frac{1}{2}d + \frac{1}{2}l.s.) = 7ch + (l.s.)^2$
$= 70 + 4 = 74$
The two calculations agree.

Sum of Big Space for this set.
Large Space + Little Space = 185 + 74 = 259
This is the same sum as the sum gotten for these values in Set one.

The values gotten for Large Space and Little Space in set two, are their average values for this Big Space.

3. Set three: Large Space befor Little Space.
Large Space extends from 3d to 4d.
$(3d)^2 = 9d^2 = 225$ and $(4d)^2 = 16d^2 = 400$.
$\underline{400 - 225 = 175 = \text{value of Large Space.}}$
$3d = 15.$ $(15)^2 = 225$
$4d = 20.$ $(20)^2 = 400$

$$\begin{array}{r} 400 \\ -\ 225 \\ \hline 175 \end{array}$$

175 numerical value of Large Space befor Little Space — agreement.

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Section 32–cont.:
Set three – cont.

Large Space befor Little Space.
The centerpoint of the Large space in this position is $(3d + \frac{1}{2}d) = 3\frac{1}{2}d$.
$2d(3\frac{1}{2}d) = 7d^2 = 175$ = numerical value.
Thus we have three calculations of the Large Space in this position in agreement.
This is the minimal value for the Large Space which occurs in this Big Space.

Little Space extends from $4d$ to $(4d + l.s.)$.
abstractly: The centerpoint $= (4d + \frac{1}{2} l.s.)$.

$$2 l.s. (4d + \frac{1}{2} l.s.) = 8 ch + (l.s.)^2$$
$$= (80 + 4) = 84$$

numerically: $4d = 20$, $(4d + l.s.) = 22$.
$(20)^2 = 400$. $(22)^2 = 484$

$484 - 400 = 84$ = numerical value of Little Space in this position. The two calculations are in agreement. This is the maximum value of the Little Space which occurs in this Big Space.

Sum of Big Space for set three.
Large Space + Little Space $= 175 + 84 = 259$
Thus the sums gotten for Big Space in all three sets are the same.

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Section 33 :

September 3, I934.

Insertion of an (l.s.) difference in an arithmetical series of d differences.

Let (l.s.) = 2 and d = 5 .

The operation of this diagram is really quite simple. Beginning at zero, the point of origin, this (l.s.) difference is the sum of (l.s.)^2 plus ~~a ll ch. suntil~~ all ch. s until the point of insertion is reached .

From this point on all d differences have 2ch. added to them.

Thus: Ifthe (l.s.)difference is first, the difference equals (l.s.)^2 ; and all d difference have 2 ch. added to them.

If the (l.s .) difference is following the I d^2 difference it equals (l.s .)^2 + 2 ch. ; and all d differences following have 2 ch. added to them.

If the (l.s.) difference follows the 5 d^2 difference it equals (l.s .)^2 Plus 6 ch. ; and all d differences following have 2 ch. added to them.

If the (l.s .) difference follows the I3 d^2 difference it equals (l.s.)^2 plus I4 ch. .

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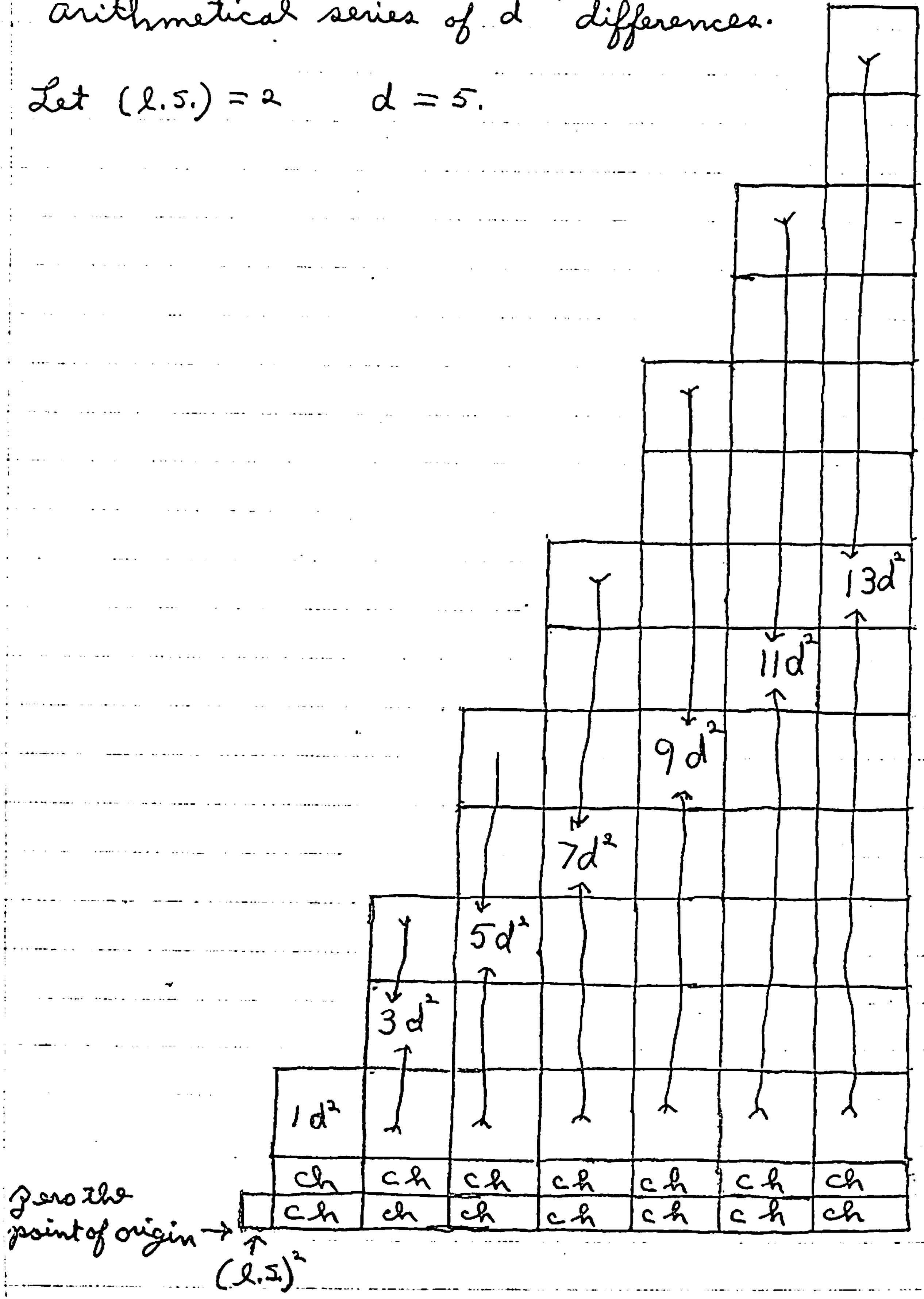
Page 13.a

Section 33:

September 3, 1934

Insertion of an (l.s.) difference in an arithmetical series of d differences.

Let (l.s.) = 2 d = 5.



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Section 3 4 :
The Triangular Series.

October 7, 1935

The right angled triangle a b c Equals square abfe in area. An arithmetical series of differences can be diagramed as a triangle just as correctly as in a rectangle.

In the illustration, triangle a D E equals triangle F D C because right angle D F C equals right angle a E D, and angle a D E equals angle ErD c (verticle angles). AS the homologous angles are equal the tria ngles
Fdc ~~triangles~~
are simular. But side FD = side DE. Therefor the two triangles are equal.

Going to the (I/2 d^2)differences:

They are presented as triangles. The top difference = I $_2$(I/2 d $)^2$, the second difference = 3 (I/2 d $)^2$. The total = 4 (I/2 d $)^2$.

Call the hypotenuse aD c the line of slope of the series. Notice that the series can be changed from a square to a triangle by shifting triangle ~~and~~ A E D to position C F D. Instead of graduated differences,
F
step by step; we have the line of slope as the upper boundary, which is even, smooth, and continuous. With this kind of boundary it is much easier to insert differences having different widths into the series.

Notice that while the base of triangle ABC ~~is~~ equals I d, the height, b c = 2 d. Thus the diagram is prepared to receive the common difference of differences, 2 d^2, to the left of b c, to form the succeeding difference 3 d^2.

The trigonometic function of angle bac is tangent = $\frac{\text{Side opposite}}{\text{side adjacent}}$ = 2/I .

The table of natural Sines, Cosines, Tangents, etc. give this angle as 63° 26+ M. The Tangent of angle BCA = I/2. The table gives the angle as 26° 34 – M.

63° 26 + M.
26 ° 34– M.

&
89° 60 M.
Equals 90°.

These angle values are the same for all triangular series.

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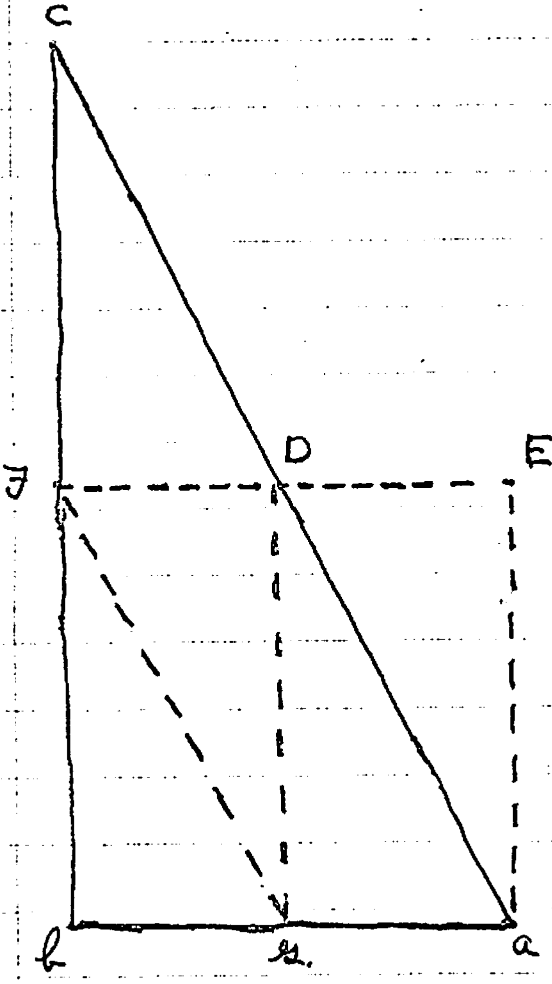
October 7, 1935

Page 14.a
Section 34:

The Triangular Series

I will give some illustrations to make plainer my ideas on changing from differences of one d value to another difference of (l.s.) value in a series. This illustration shows a series of only $1d^2$ value. I present the increase up to 1d both in $(d)^2$ and in $(\frac{1}{2}d)^2$.



line ba = FE = 1d.
bc = 2d
aE = bF = 1d.
aG = Gb = FD = DE = ½d

a proposition from plane Geometry states: "The lines joining the middle points of the sides of a triangle divide the triangle into four equal triangles". — Plane Geometry by G. A. Wentworth, Page 69, Exercise 49.

The line FD divides the triangle abc, horizontally, and the line DG divides the triangle abc vertically.

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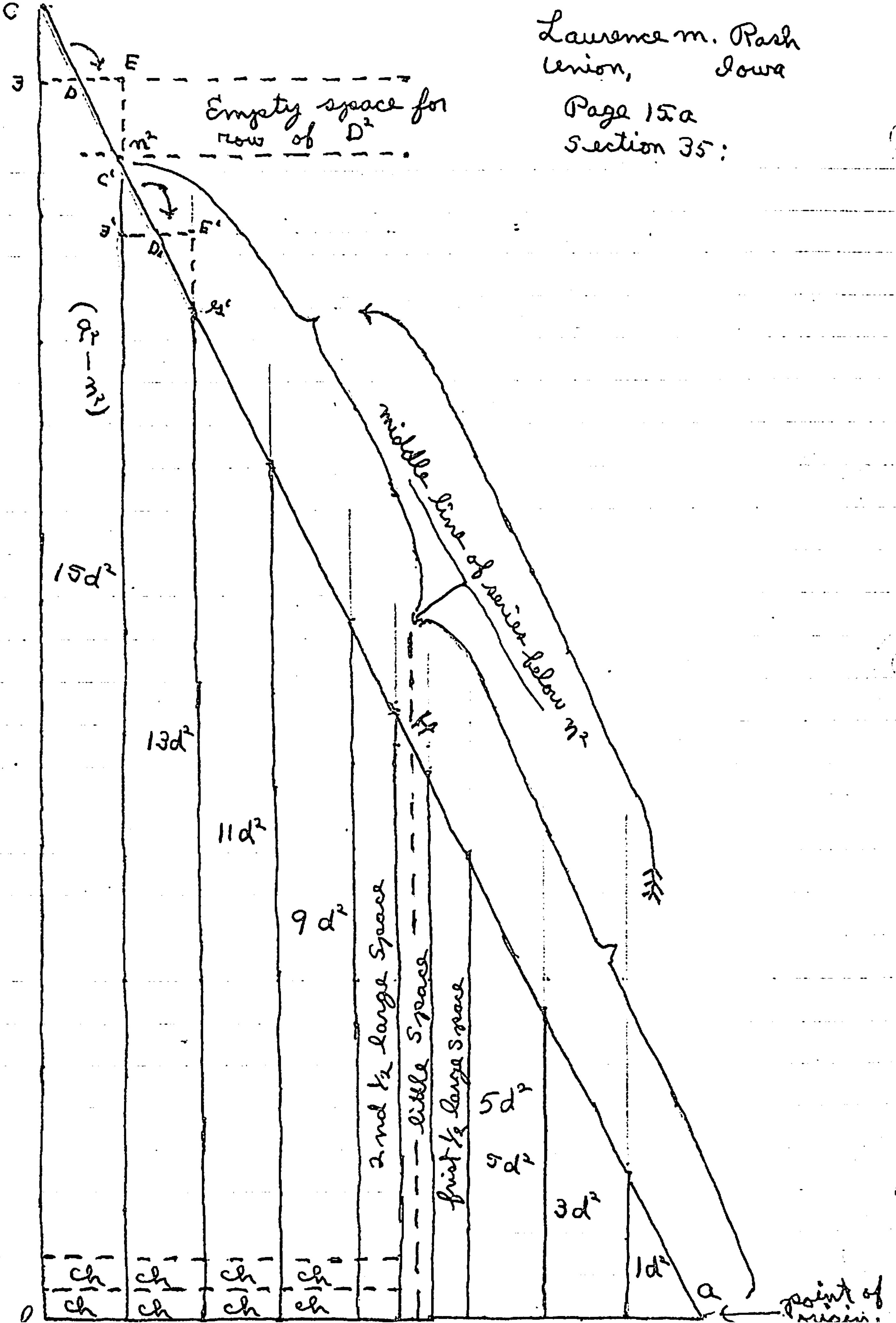
Page 15a
Section 35:



Comparision between illustration and complete diagram.

The diagram page I 0.a, Sections 22 and 23 -- Using the arrangement of the Big Space of Set 2, Section 32, Page I2. b; the series of differences of the diagram and the illustration are identical.

The diagram is distorted especially vertically so as to fit conveniently into the other material on the page. I have not drawn the line of slope, becauseof the diatortion it would not pass thru the ends of the differences correctly. The point of origin would be the upper left hand corner of I D^2. Because of the row of d^2 s it is not easy to show exactly where the series of differences hinges -- there is a definite variation between the illustration and the diagram.

While the values of this illustration may be correlated with the values of the diagram; yet there is a fundament difference between them. This illustration has no row of d^2 s. After the ~~frist~~ first half of the series below N^2 is hinged over on top the last half. we do not have divisions, but rather sums of two differences. Because there is no row of D^2 s, this folded series is still longer below N than the diagram. While this illustration is not in the direct line of the solutiom of the square root, I feel that it answers many questioms concerning just how small differences are inserted in the series.

I have tried to draw this illustratiom to scale: (l.s.) = 2/8 of an inch; d = 5/8 inch. I have used the same values as in the diagram, and the two drawings present the same facts. Both A^2 and N^2 of one equal those of the other.

If triangle C F D is moved to the position of triangle E D C' the ($A^2 - N^2$) space is levelled off. If triangle C' F' D' IS MOVED to tri angle E' D' G' the next space below N^2 is levelled off. This makes it easier to see the correlation between the two drawning; ~~to be~~

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A better arrangement of the complete diamgram — February 3, I936.

Review of my method for extracting the square root of an N^2.

Article I.

Page5 — Section I0 — A suitable A^2, whose square root is known is chosen. Thus the last Space or difference $(A^2 - N^2)$ is set up.

Article 2.

Page 6 — Section I2 — The diagram set up Section I4. It is impotrtant to complete the fractional division farthest to the right ,so as to be sure how many d differences it contains. When this is known, the number of d differences below N^2 is known abstractly. — In other words N^2 can be calculated in D^2 value. Also the $(A^2 - N^2)$ space is known abstractly in D^2 value. These two facts being known the Integral diagram can be set up.

Page 7, Section I5 — Again it is shown how important it is to complete the calculation of the number of d differences in the fractional division farthest to the right.

Article 3

P.I0 — Sections 22 and 23 — The complete diagram is set up numerically.
Section 24 — Only one full difference in the fractional division to the right.
Section 25 — The $(A^2 - N^2)$ difference is knownin D^2 s.
Here also the Integral diagram can be set up.

Article 4 — On Page Io and following Pages we proceeded with calculation of the parts of the " Big Space ". It is necessary that the fundamental reasons for these operations be understood.

However ther are two faults with this operation. (I) They are slow, laborious, and clumsy; (2) If the numerical value of N is unknown,it is impossible to separate the value of the " Little Space " from ch values, as they are included togerther in the row of D^2 s. (section 30, bottom page II.a) — I ch ÷ 27/395 ch to be taken from the row of D^2 s.

Section 37: — Abetter arrangement of the complete diagram.
Article I.

There is another arrangement of the complete diagram which will solve these difficulties.

Superimpose the Integral diagram on the complete diagram and subtract it from the complete diagram. This will leave the "Little Space" in a column to its self to the far right of the diagram. I call this column the (l.s.c.) — THE Little Space Column. This will place the Little Space in a fourth position in the "Big Space". The two rows of ch under the Integral diagram are attached permanently to their respective divisions.

This is a "break thru" almost equal in value to the "Big Space break thru". I was first able to do this February 3, I936. — After about eight years work on this problem.

Article 2. — A step by step summary of procedure.

Article 2. — Astep by step summsry of procedure.

Step I. Choose a convient A^2, alittle larger than N^2 , Whose square root is known.

Step 2. Subtract N^2 from A^2 and find ($A^2 - N^2$) numerically.

Step 3. Divide N^2 by ($A^2 - N^2$), numerically, and set up the complete diagram. ~~Tis~~ This having been done the differences will be known abstractly in d^2 s, until the Big Space is reached.

Step 4 . — To calculate the number of full d differences in the sum of the fractional division to the right of the diagram plus row of D^2 s .

There are five possibilities:

(a) — There is only a Little Space. In this event the v below N^2 will be an even number. (That is the number of full d differences below N^2 will be an even number.)

(b) — There is only a full d difference. In this caee,the v below N^2 will be an odd number. The diagram will be a Integral diagram.

(c) — There are a full d difference plus a Little Space. The v below N^2 is odd.

(d) — There are two full d differences below N^2. The v below N^2 is even. This is an Integral diagram.

(e) — There are two full d differences plus a Little Space. The v below N^2 is even.

Procedure: After the top row of differences, the next full difference is known abstractly in D^2 s. Calculate the top part of the fractional space at this rate. Add the value of the row of D^2 s. If this sum does not equal the next full d difference, then possibilities (d) and (e) are eliminated. For clearly the bottom fractional space can not equal a full d difference.

Calculate both the top and bottom part of the fractional space at the first difference rate, and add the value of the row of D^2. Determine whether the sum is a Little Space, a full d difference, or a full d difference plus a Little Space.

Trial 2. If the calculation of the top part of the fractional space , plus value of row of d^2 s. exceeds one full d difference by a considerable amount; the bottom part of the space must be calculated at the second higher difference rate. Then see if there is enough D^2 value in the row of D^2 to complete both of these two fractional differences. It will be easy to see if there is a Little Space.

It is very important that these calculations be correct befor proceeding further.

Page 18.a
Section 37.

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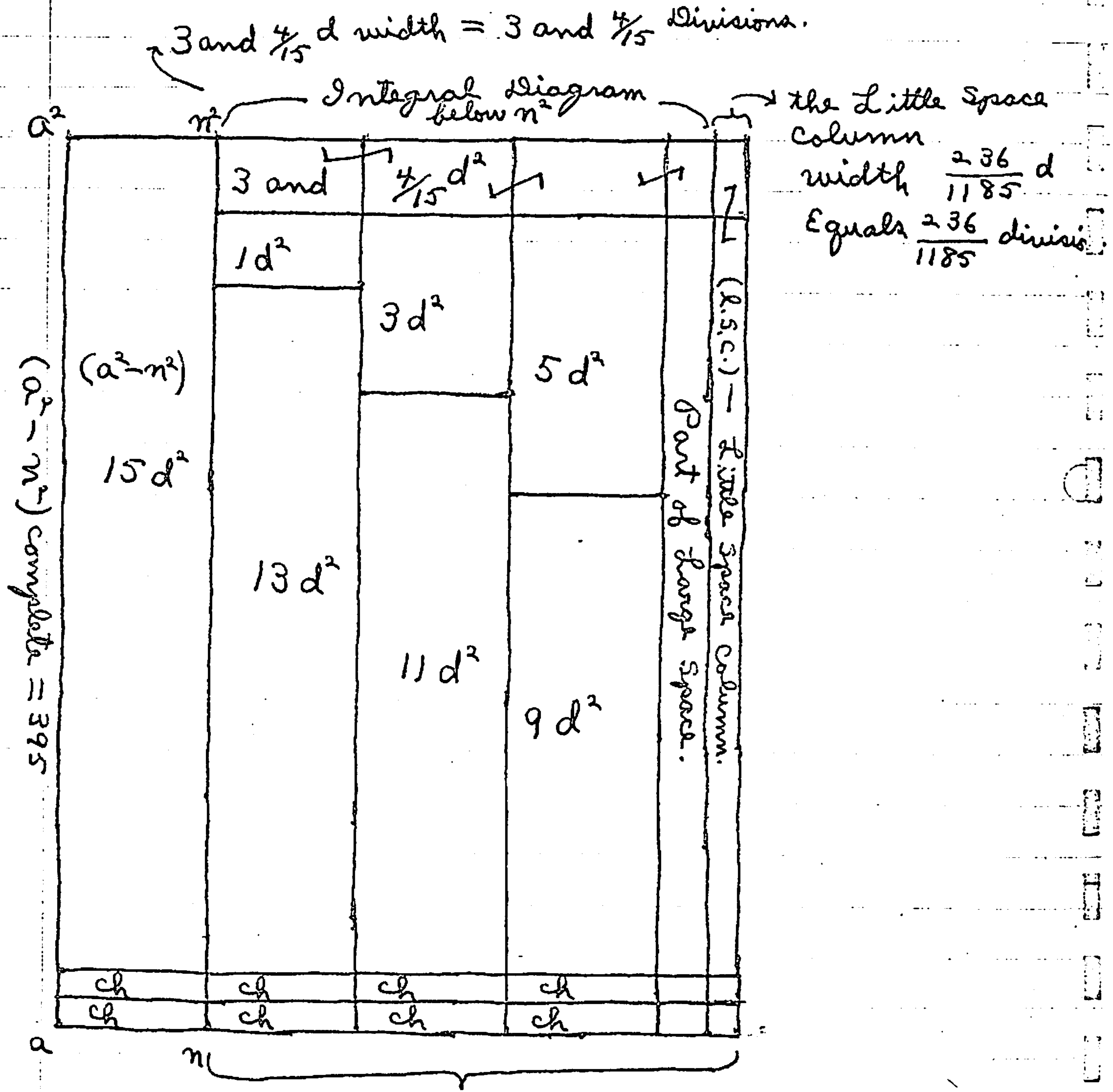
The Integral Diagram Superimposed on the Complete Diagram and Subtracted from it.



Complete Diagram below N^2.
3 and $\frac{184}{395}$ d width.
3 and $\frac{184}{395}$ divisions below N^2.

Page 18. b
copy page 10.a
Section 37:

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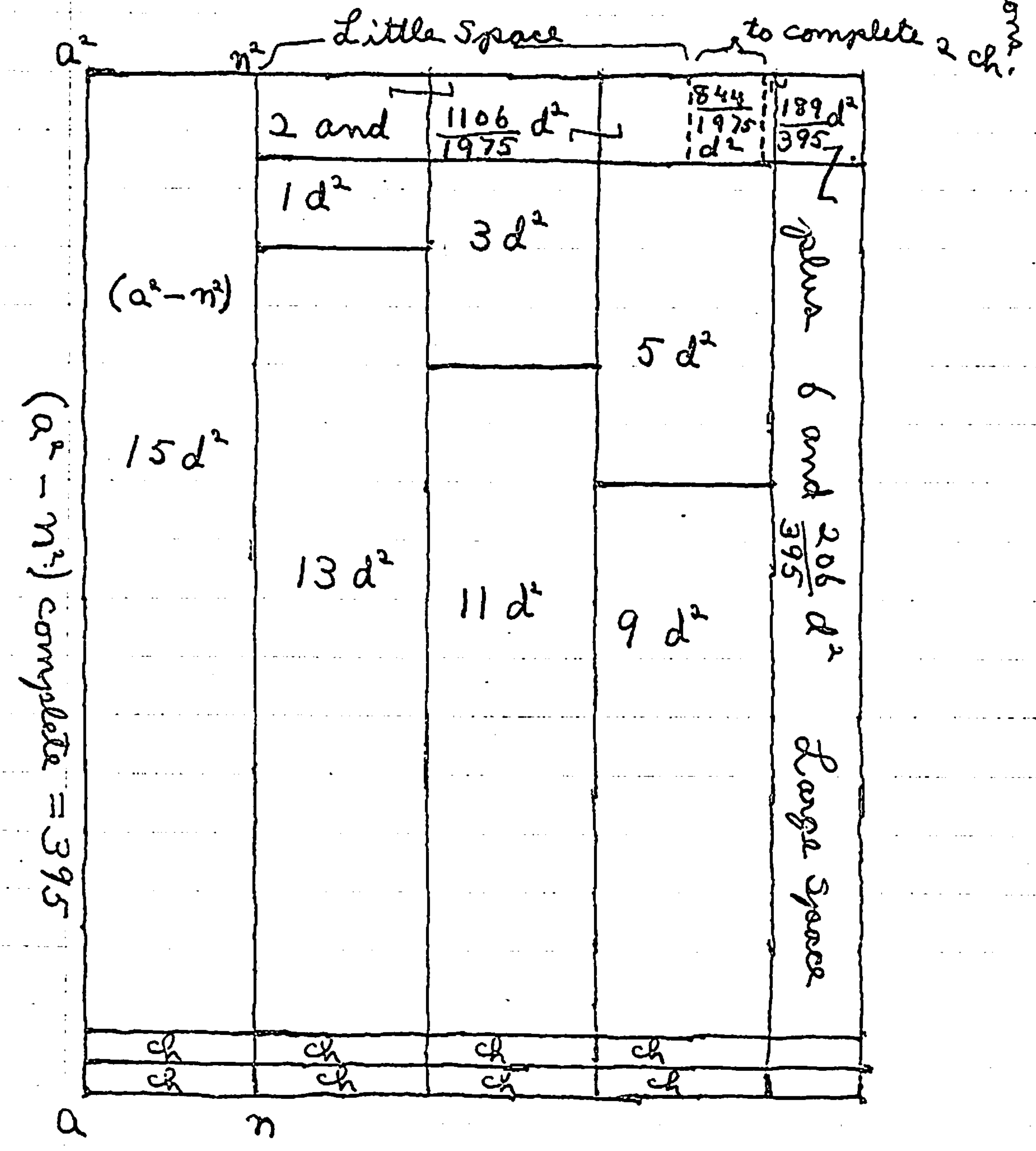
March 5, 1932

The "Big Space" idea

Let a = 42, then $a^2 = 1764$
Let n = 37, then $n^2 = \underline{1369}$
Subtracting $(a^2 - n^2) = 395$

$$395\overline{)1369}(3$$
$$\underline{1185}$$
$$184$$

There are $3\frac{184}{395}$ divisions below n^2.



$$ch = (d \times l.s.) = (5 \times 2) = 10$$

Article 2.

Step 5. When the number of full d differences below N is known, we can calculate N^2 and $(A^2 - N^2)$ in D^2 s.

Step 6. Set up the Integral Diagram by dividing N^2, known in D^2, by $(A^2 - N^2)$, known in D^2, This calculation is done in abstract values.

Step 7. Placing the two $(A^2 - N^2)$ spaces so that they coincide, subtract the number of divisionsin the Integral Diagram from those of the complete diagram. The value left is the width of L.S.C. in d value.

Note: These operations can be done even when making calculations for a surd.

EXPLANATIONS

On Page I8.b, I have given a copy of Page I0.a, for the convience of my readers. Please note that the I89/395 D^2 added from the rowof D^2 s is 5/395 D^2 more than the I84/395 D^2 in the block at the top of the fractional division on the right of the diagram. The 5/395 D^2 value comes from added space next to the left in the row of D^2 s.

Facing this diagram is the better arrangement of the values of the same diagram, on Page I8.a. The advantage of this rearrangement of values, is that the "Little Space" is completely and definately separated from the rest of the diagram in (L.S.C.) to the far right. This calculation is just as effective when making calculations for SURDS.

Page I8.c contains two calculations:
(I)- IS THE WIDTH OF (L.S.C.) in d value.

(2) - Is the calculation of the parts of the 4 th. difference. That the sum of parts equals 7 D^2 is proof that the diagram is constructed correctly. Please note again that the "Large Space2 and the "Little Space2' are completely isolated from each other.

Also note that (L.S.C.) is inserted in the arithmetical series and in the "Big Space" at anew point. I shall deal with this in the next chapter.

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Page 18.c
Section 37:

Calculations for constructing the Integral Diagram. to be Superimposed on the Complete Diagram of Page 10.a — There is a copy of Page 10.a on Page 18.b.

(1) It has been proven that there is only one full d difference between the top and the bottom series of differences. Therefor n contains but 7, d differences (in addition to l.s.). Therefor the integral series below $n^2 = 49\ d^2$; and $(a^2 - n^2)$ has 15 d^2.

$15)\ 49\ (3$
45
4

There are 3 and $\frac{4}{15}$ divisions in the integral diagram below n^2.

The complete diagram = 3 and $\frac{184}{395}$ divisions.

$395 = (5)(79)$
$15 = (5)(3)$
Least common multiple equals $(5)(3)(79) = 1185$.

Subtraction:

Complete diagram — $3\ \frac{184}{395} = 3\ \frac{552}{1185}$ divisions

Integral diagram — $3\ \frac{4}{15} = 3\ \frac{316}{1185}$

$$\begin{array}{r} 552 \\ -316 \\ \hline 236 \end{array}$$

d width of (l.s.c.) $= \frac{236}{1185}\ d = \frac{236}{1185}$ division.

(2) Calculation of the parts of the 4 th. difference.
The 4 th. difference has $7d^2$ in the Integral series.
The d width of column for Part of Large Space is $\frac{316}{1185}$ or $\frac{316}{1185}$ division. $7 \times \frac{316}{1185} = \frac{2212}{1185} = 1$ and $\frac{1027}{1185}\ d^2$.

First fractional 4 th. Space = 1 and $\frac{1027}{1185}\ d^2$
From row of d^2 = 3 and $\frac{316}{1185}\ d^2$
2 nd. fractional 4 th. Space = 1 and $\frac{1027}{1185}\ d^2$
Total 5 and $\frac{2370}{1185}\ d^2$

$$\begin{array}{r} 1027 \\ 316 \\ 1027 \\ \hline 2370 \end{array}$$

$1185)\ 2370\ (2$
2370

$(5+2) = 7\ d^2$ agreement

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Addendum to Section 37; Article I. :

There are Two important calculations of the values of this "Little Space column" created under this Section and Article:.

(I). A-s its width is known in d, its value can be calculated abstractly in values of D^2 And ch.

(2). As its width also is known in d, its width is also known as a fractional division. From the $(A^2 - N^2)$ space value,its numerical value is known exactly.

This separation of its value from the value of the arithmetrical series of d differences is a very important "break thru".

Note:

The numerical values in column 4 of the table at the top of page 3.c — Section 6form an arithmetical progression in which I8 is the common difference between consecutive terms.

The abstract values in D^2 units of column 4 of the table in the middle of page 3.c, Section6, form an arithmetical progression in which $2D^2$ is the common difference of consecutive terms.

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Page I8. d .a.
Section 37 end I/4.

A corollary of the Illustration on Page I5.a is that any horizontal segment ofline BA is a measure of the value in d of differences of squares above it. That is whether it is a full d difference, a fractional d difference, or a (l.s.) difference, etc. If the segment is I/2d, then the difference i s I /2d of a full d difference. I f the segment should be 3 /7 of a full d, difference, then the difference would be 3/7 of a full d difference. This reasoning applies only to triangular series.

In a diagram the relationship is changed because of the row of D's across the top of the diagram. In a diagram these fractionals apply to each menber of the sum of two fractional differences, which occur together in the last di ision to the right befor (l.s.c.). The fractional d width of a division is not a measure of the d difference it contains, but a fractionalof thethe value of the entire large space. I am speaking of the integralseries of differences.

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Page I 8.e
Section 37 and I /2 :

Calculating the Centerpoint at which (L.S.C.) is inserted in the Arithmetical ~~serirs~~ series of differences.:

The insertion of an (L.S.) difference into a arithm-etical series of (d) differences is explained in Sestion 3 3 , Pages I 3 and I 3. a. It is explained that in the illustration, if the (1.s.) difference follows the seventh (d) difference, its value would be (1.s.)^2 + I 4 ch.

Now going to the illustration under Section 37, page I8.a. If the (1.s.) difference is inserted just below N^2, it would equal [(1.s .)^2 ~~Each~~, + I 4ch,] because it is inserted following the seventh d difference. This value is also the sum of all ch under the Integral diagram below N^2 plus the (1.s.c.). Therefor subtracting all ch under the Integral diagram below N^2 from this sum gives (1.s.c.)as a remainder.

As explained under Section I8, Rule 2, Page 8 "any term in the same series equals 2d times its centerpoint". Therefor divide the abstract value of (1.s.c.) by 2(1.s.); the quotient will be its centerpoint.

Another method: Subtract the number of d differences (including the fractional of the large space) in the bottom row of differences from the sum of all d differences below N^2. Thus the number of d differences which ~~come befor~~ come befor the position of (1.s.c.) is known. Of course the centerpoint of (1.s.c.) would be I /2 ~~greater than that~~ (1.s.) centerpoint. greater than that.

RESTATEMENT OF SEC. 37½
The Little Space and the 2 rows of ch are a sum. Sept. 3, I 934 .

Under Section 33, Page I 3 it is shown that (1.s.)^2 + (the number of ch s) is a sum. This sum is the same whether the (1.s .) value is inserted ~~thefor~~ thearithmetical series of d differences , ~~as en~~ or as an (1.s.c.) anywhere up to the position just below N^2. There are two calculations of the abstract value of (1.s .c.).

(I). Under Section 37½ it is shown that the d width of the fractional space to the right of the bottom row of arithmetical differences in the integral diagram is a measure of the fractional of the large space which completes the bottom row of differences. The (1.s.c.) is inserted just befor this fractional difference. Therefor subtracting the number of differencrs of thebottom row of of the arithmetical series of the integral diagram, from the entire number of differences, gives the number of d differences befor the insertion of (1.s.c.). Therefor [(1.s.)^2 + 2 (number of differences befor the insertion) ch] = the abstract value of (1,s, .c.) .

(2). Calculate the value of (1.s.)difference just below N^2. Subtract the number of ch beneath the integral diagram. The remainder equals (1.s.c.).

Page I8 and I/2
Section 37 and 3/4
Inserted near complition of this book.

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Calculating Integral Diagrams — Page 6.b — Section I4.

I — Restatement of Problem: ~~Section I4~~ .

Our only known diagram, when working with a set of values of which N is unknown; is the complete diagram, expressed in numerical values for the divisions. The only way to calculate the Integral Diagram is in abstract values. We can not calculate the abstract value of the ($A^2 - N^2$) space, until we know how many full d differences there are in the division furthest right plus the row of d^2 s.

Note: The reason 844 / I975 d^2 is subtracted from the row of d^2 s (Page I0.a and Page I8.b) is that we are working with a complete diagram, without subtracting the Integral Diagram. — Explained (Page I0.a (2 ch needed to supplement minimal values of spaces) and (Page II.b, section 3I, item (2) value taken to complete 2 ch) —

II — A Master Plan to Determine How many full d differences of the arithemetical Series (Page 4 — note bottom of page) are contained in the sum of the row of d^2 s plus the division furthest to the right, whether it be an entire or a fractional division. It is neccessary to know this in order to determine the ordinal number of the ($A^2 - N^2$) difference. Knowing this we can calculate the Integral Diagram.

There are two considerations:

(I). The sum of the row of d^2 s increases one for each consective division. The consecutive d differences of the arithemetrical series increase $2d^2$ for each division. Therefore the sum of the row of d^2 s would equal approximately I / 2 a difference in the right division.

(II). The numerical coefficient of the division to the right, is also the coefficient of the parts of differences contained in the spaces in both the top and bottom row of differences of this division. Both spaces are below the row of d^2 s. These are the higher end of the top row of d differences; and the beginning of the lower row of d differences. We are only interested in completing the d difference series, at this moment. The value of any difference equals $(2v - I)d^2$.

Case I — In the present complete diagram for the $\sqrt{8}$ (Page 43), there are eight full divisions. That means that there are two full d differences in the division to the right. The sum of the row of d^2 s = (l.s.c.) .

Case II — If the division coefficient were, very near or, a little over 3 / 4, the two fractional spaces could still be completed into two full d differences by addition of value from the row of d^2 s.

Case III — If the division coefficient were approximately one half, there would be only one d difference.

Case IV — IF the division coefficient were very small, there would be only an (l.s.c.) space.

Case V — In marginal cases the actual completion of the differences by addition from the total value of the row of d^2 s must be did so as to be sure.

We are now able to determine the ordinal number of the ($A^2 - N^2$) difference, and thus are able to set up the Integral Diagram.

BARRIERS

There are Barriers between the dimensional Continua which can not be crossed. (I use the word "continuum" in the same sense as ALBERT EINSTEIN on page 65, Minkowski's Four-dimensional Space, The Special Theory Of Relativity). For instance, an unit of measure of the first dimensional continuum is one inch. This brings up the question, "How many linear I inch lengths lain side by side would equal I squareinch?" The answer is that an infinite number would make no progress at all. To enter the Second dimensional continuum we must have a surface unit given. YOU CAN NOT CROSS A DIMENSIONAL BARRIER.

However values in the first dimensional continuum do effect values in the second dimensional continuum; and vice versa. Thus: The values of d and (l.s.) effect the values of d^2 and $(l.s.)^2$ respectively. The values of both d and (l.s.) effect the values of ch. You must find a relationship which already crosses the dimensional barrier.

Also, when N is unknown, there are intra-dimensional barriers. There is a barrier between the values of d and (l.s.) —that is the ratios of their values is unknown. In the second dimensional continuum the ratios between the values of $(d)^2$, $(l.s.)^2$, and ch, are unknown. If the ratio of the values in either column is solved, then continuum is solved, then the ratios in the other continuum is disclosed. These inter dimensional relationships are similar to functions. My solution of this square root problem is reached by approaching an intra-dimensional barrier.

It is possible in a number of ways, to construct complete diagrams which are closely related. I tried for a number of years to construct a complete second diagram from the first complete diagram. I would almost finish the calculation of the second diagram, but there would always be a minute incalculable excess or deficiency. (That is using the proceedure I would have had to use with a surd.) My plan was to compare the two complete diagrams, much the same as two simultaneous equations, and find the value of unknown parts.

(This was very interesting, but it did not promote the extraction of the square root. The work was very voluminous. It's inclusion would increase the size of this book about three times. Therefor space and the patience of my readers forbids it.)

The nature of this barrier was not that there was any obstructiom in the way, but always there was a little information missing which was neccessary to a numerical solution. Oddly enough, I could always get an abstract solution.

Time and again it occured to me that an intelligence of a higher dimensional continuum than human, (Therefor of a higher level of intelligence than human), would have the needed imformation at his command . than

I quote Shakespeare's Julius Caesar, Act. I , Sc.2 Line I34:

"The fault, dear Brutus, is not in our stars,
But in ourselves, that we are underlings."

Chapter V

Early "middle Age"

Page 20 to Page 21. b
Section 39 to Section 40

Laurence M. ~~Raugh~~ Rash
Union, Iowa

Chapter V

The G space isolated –

The number of G s in (l.s.c.) calculated without extending the calculations to values inside the G space.

Section 39: April 27, I 937.

This Section shows a rearrangement of the values composing (l.s .c.) . The frist diagram on Page 20.a is the casual arrangement of (l. s.c.). The second diagram shows the rearrangement of values of (l.s.c.), placing 84/I3 ch entirely above the 2 rows of ch.
August 20, I937 (rewritten) Values of the surfaces.

(ch) = (d height) (l.s.) width.
(l.s .)^2 – (l.s. hei ght) (l.s.) width.

The height of the diagram above the two rows of ch is known in values of d . Therefor the number of ch in the (l.s.c.) arranged in a rectangFeular column whose side is this d height, will give its width in (l.s.) values.

EXPERI-MENTATION WITH A NUMBER OF DIAGRAMS SHOWS THA-T THE COEFFICIENT OF THI-S (L.S.) WIDTH IS ALWAYS A LITTLE LESS THAN I /2.

There can be no variation of this rule with diagrams developed as I have developed them, because this method is capable of finding square roots whose errors are only inffni-tesi-mal.infinitesimal. Infinitesimal is defined by Webster's Collegi-ate Dictionary (year I 9I 8) a s "2. I mmeasurablyor- or incalculably small; very minute."

This rule is very important because the developement of the rest of the soltion depe-nds upon it.

Note: If some of myreaders have a feeling of unsureness about the soundness of this proof of this rule; allow me to point out that the cause may be , we are approaching near the intra– dimensional barrier between the value of d and ,of (l.s.). The actual ratio between the two is, in many cases, just beyond the grasp of the human mind.

When this column is projested across the two rows of ch we get (z (l.s.) height) ((less than I/2 L.s.)width), – the product is (less than (l.s.)^2 equal to S. This leaves the G space equal to a small fraction of (l.s.)^2 .

Page 20.a
Section 39:

Laurence M. Rash
Union, Iowa
Nov. 10, 1936

I Diagram showing the frist arrangement of (l.s.c.).

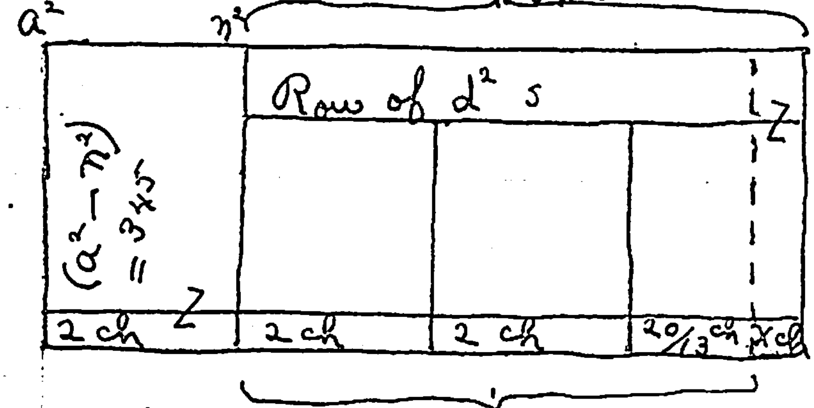
Let $d = 5$; v of $n = 6$; and $l.s. = 2$

Then $n = 32$, and $a = 37$

and $n^2 = 1024$; $a^2 = 1369$

and $(a^2 - n^2) = 345$.

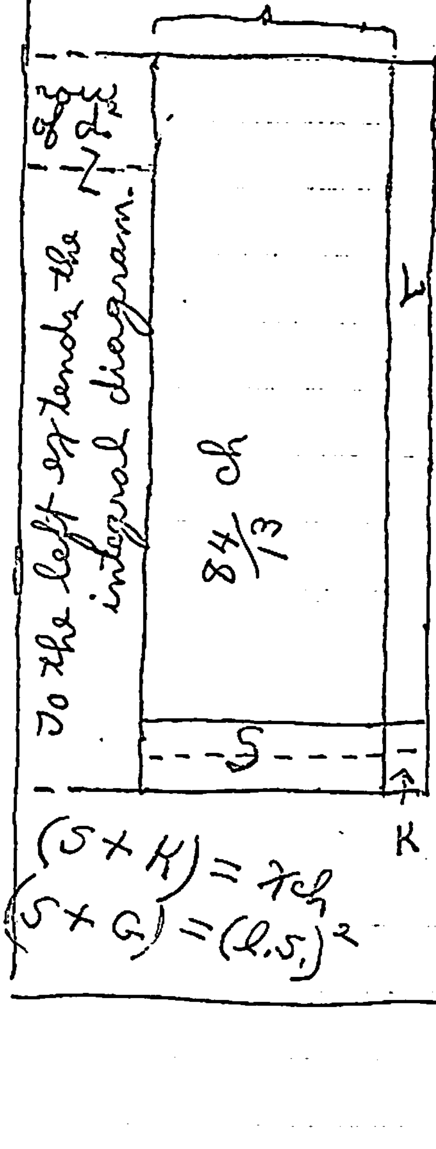
Complete diagram below n^2 equals 2 and $\frac{334}{345}$ divisions



Integral digram below n^2 equals 2 and $10/13$ divisions

Total (l.s.c.) above 2 rows of ch equals $(l.s.)^2 + 6$ and $\frac{286}{(13)(345)}$ ch.

II. Diagram showing only the second arrangement of (l.s.c.).



$(S + K) = 7ch$
$(S + G) = (l.s.)^2$

The G space = $1/169$ (l.s.) = (L + K)

I Calculation of a^2

$$\begin{array}{r} 37 \\ 37 \\ \hline 259 \\ 111 \\ \hline 1369 \end{array} = a^2$$

Calculation of n^2

$$\begin{array}{r} 32 \\ 32 \\ \hline 64 \\ 96 \\ \hline 1024 \end{array} = n^2$$

Calculation of $(a^2 - n^2)$

$$\begin{array}{r} 1369 \\ -1024 \\ \hline 345 \end{array}$$

Calculation of number of divisions in complete diagram below n^2

$$345\overline{)1024}\ (2 \text{ and } \tfrac{334}{345} \text{ divisions.}$$
$$\begin{array}{r} 690 \\ \hline 334 \end{array}$$

Laurence M. Rash
Union Iowa

Page 20.b
Section 39:

Calculation of number of divisions in integral diagram below n^2.
$(a^2 - n^2)$ is the seventh difference and contains $13d^2$ above the 2 rows of ch.
n contains $6d$.
n^2 contains $36d^2$.

$13 \overline{)36}$ (2 and $\frac{10}{13}$ divisions
$\underline{26}$
10

Calculation of the value of (l. s. c.).
There are $5\frac{7}{13}$ ch in 2 rows of ch beneath integral diagram as there are $6d$ in n, there are $(l.s.)^2 + 12$ ch in the (l.s.) space just below n^2. By subtraction of ch, there are $(l.s.)^2 + 6\frac{6}{13}$ ch in the l.s. column.

Checking the value of (l.s.c.) by it's centerpoint.
The integral diagram is $2\frac{10}{13}$ d long.
$2\left(2\frac{10}{13}d\right) = 5\frac{7}{13}d$ = the length of the 2 rows of differences.
There are 6 d below n. Then $6d - 5\frac{7}{13}d = \frac{6}{13}d$.
$\frac{6}{13}$ d is the measure of a fraction of a division having surface equal to the row of d^2 s.
$2\frac{10}{13}d + \frac{6}{13}d = 3\frac{3}{13}d$ = lower side of (l.s.c.).
Then $(l.s.c.) = (l.s.)^2 + 6\frac{6}{13}$ ch. check

Checking the value of xch — fractions only
The complete diagram = 2 and $\frac{334}{345}$ divisions $= \frac{4342}{(13)(345)}$ division.
The Integral diagram = 2 and $\frac{10}{13}$ divisions $= \frac{3450}{(13)(345)}$ divisions.

$$\begin{array}{r} 4342 \\ -\ 3450 \\ \hline 892 \end{array}$$

$\frac{892}{(13)(345)}$ = d width of (l.s.c.)

$2\left(\frac{892}{(13)(345)}\right)$ gives $\frac{1784}{(13)(345)}$ ch = xch

---

Then $\left[(l.s.)^2 + 6\frac{6}{13}\text{ ch}\right] - \frac{1784}{(13)(345)}\text{ch} = \left[(l.s.)^2 + 6\frac{286}{(13)(345)}\text{ch}\right]$ equals the (l.s.c) above the 2 rows of ch.

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Section 39:

II Calculations for diagram showing only the second arrangement of (l.s.c.).

There are 6 ch + $6/13$ ch in (l.s.c) = $\frac{84}{13}$ ch in (l.s.c). Place all the ch in (l.s.c.), above the position of the 2 rows of ch; in (l.s.c). This column is 13d high — the same height as $(a^2 - n^2)$. Now a ch surface has one side equal to d, the other side equal to (l.s.). The product of the sides equals the surface.

Now if these $6\,6/13$ ch are rearranged so that the column has 13d as one side, the width will be less than befor.

$\frac{84}{13} \div 13 = \frac{84}{169}$. $\frac{84}{169}$ (l.s.) is the width of the column.

$\frac{84}{169}$ (l.s.) width across the 2 rows of ch s gives $\frac{168}{169}(l.s.)^2 = S$.

The remainder of $(l.s.)^2$, $1/169\,(l.s.)^2 = G$

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Section 40:

Feb. I 2, 1938.

Article I . On Page 20. a , Section 39, Diagram II we have all the ch placed above the two rows of ch. Now if we divide the values of this column by the value of xch, we get the number of xch it contains. Now substitute the equal value (S ÷ K) for xch. We now have a column of S and a column of K above the 2 rows of ch. As the number of G in S is known for any given diagram, this column of S may be expressed in values of G units.

Article II. G is a common measure of both $(l.s.)^2$ and the column of S. Therefor, is a common measure of $(l.s.)^2$ and the 84/I3 ch in diagram II, Section 39, Page 20.a, except the small column of K is incommensurable in units of an entire G, This is the first time I have found anything like a common measure of the part of (l.s.c.) above the two rows of ch and xch.

~~[illegible] EFFORTS [illegible]~~

Article III I might as well explain now, so that my readers will understand my efforts and be able to appreciate my progress, that the aim of the rest of this solution is to obtain exactly, or more realisically, an exceedingly close approximation of the number of G in (l.s.c.). I call this value the "True Value" of (l.s.c.) or by the initials " T.V.".

It works as follows: Take the diagram on page 2I.a, Section 40; From "T .V.", subtract the lone G which is in the position of the slender column to the extreme fight of the diagram; The remainder, including column of K, is a column of uniform width. The number of G in the S across the 2 rows of ch is now definitely known for any given diagram. Its height is 2(l.s .) for all diagrams. The height of this column above the two rows of ch is definitely known in d for any given diagram. Its value in G equals ("T.V.) minus (S ÷ I) or $(l.s.)^2$.

Thus, from the values in G, we have a numerical ratio between 2(l.s.) and the number of d in the height of the diagram above the 2 rows of ch. Having a numerical ratio between the value of (l.s.) and the value of d, the rest is easy.

Page 21.a
Section 40

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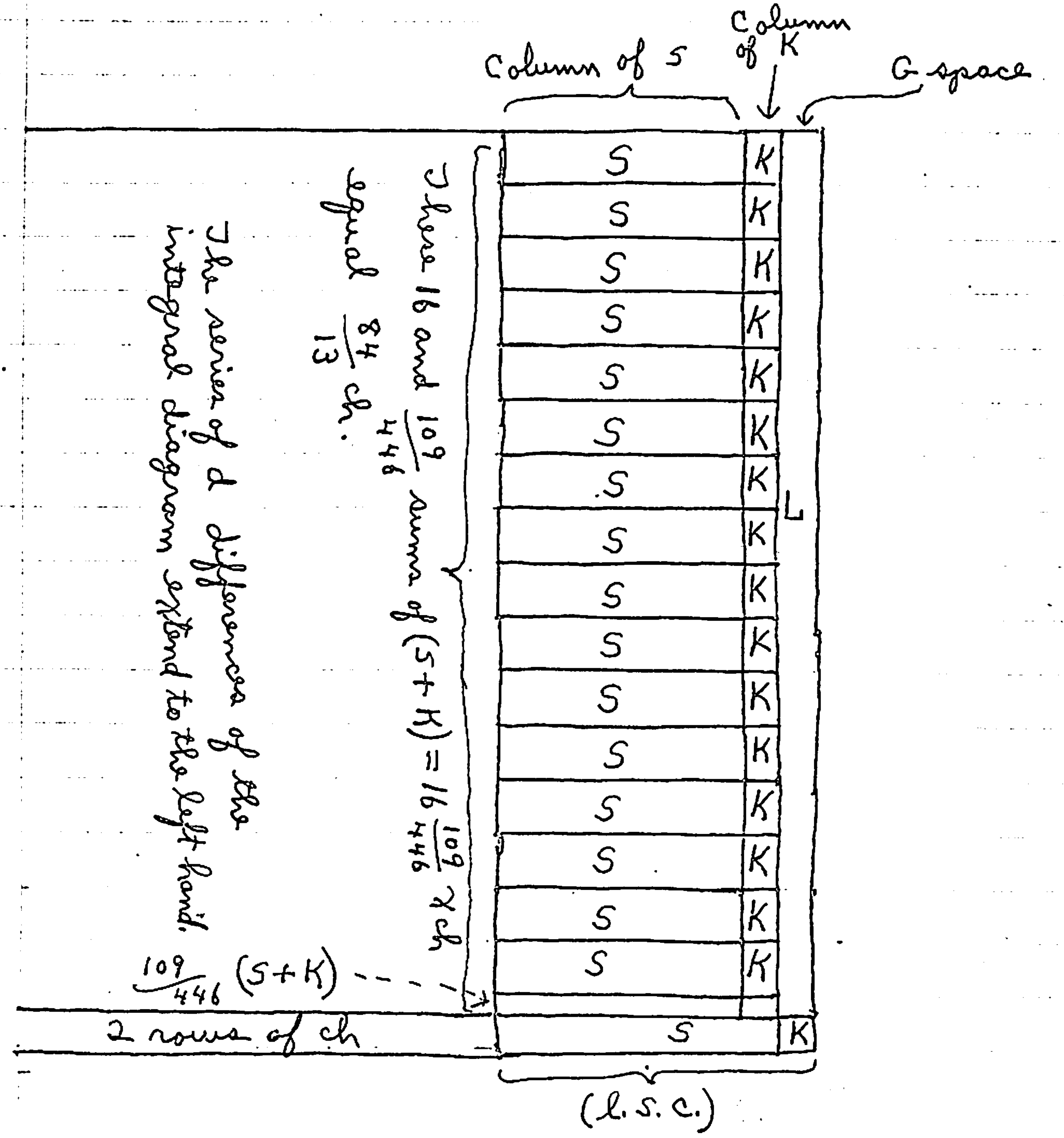
Feb. 12, 1938

III – a diagram showing a refinement of diagram II on page 20.a. Only the (l.s.c.) at the right hand end of the integral diagram is shown.



In this diagram the width of the (l.s.c.) is exaggerated in relation to it's height so that I could picture the small parts plainly.

Section 40:

From the calculations on page 20. b and Section 39, we have the values: There are $6\frac{6}{13}$ch equal to $\frac{84}{13}$ch in the (l.S.C.),

and $\frac{1784}{(13)(345)}$ ch in xch

Put them over the common denominator (13)(345).

$$\begin{array}{r} 345 \\ 84 \\ \hline 1380 \\ 27600 \\ \hline 28980 \end{array} \qquad \frac{84}{13}\text{ch} = \frac{28980}{(13)(345)}$$

Divide $\frac{84}{13}$ ch column by xch.

$$1784 \overline{)\,28980\,}(16$$
$$\begin{array}{r} 1784 \\ \hline 11140 \\ 10704 \\ \hline 436 \end{array}$$

There are 16 and $\frac{436}{1784}$ xch in this column.

Now on page 21. a, this Section, we substitute the equal sum of values, (S+K) for xch. The number of G in each S is known. Therefor, with the exception of column of K, the value of the entire (l.S.C.) is known in units of G.

Chapter VI
Late "middle Age"

Page 22 to Page 26.c
Section 41 to Section 44

Chapter VI – "LATE MIDDLE AGE"

Analysis of G Space

Section 4 I : Abstract proof:

Article I . Diagram page 22. a

Suppose all the G in (l.s.c.) to be rearranged in verticle rectangular columns which extend the entire height of the (l.s.c.) and cover all the space.

Fact I: The number of columns equals (T.V.).

Fact 2: Now (L ÷ K), equalto one G, is (I / T.V.) part of (l.s .c .).

Fact 3: Likewise L is (I / T.V.) part of all the (l.s .c.) a bove the two rows of ch.

Fact 4 :

Also L is I/ [(T. V.) − I] part of [(column of S) ÷ (column of K)].

Fact 5: And (L ÷ XCH) ≃ [(l.s. $)^2$ /T.V.] PART OF (l.s.c.).

There are corresponding relationshipes of values inside the G Space shown by this rearrangement. The G Space is a minature of (l.s.c.). THe height of both are the same But the width of G ≃ I / T.V. that of (l.s.c.).

Fact 2.I : Take afractional value of L equal to (I / T.V.) part of L.Call this value ÷L Rule : There are as many ÷L in L, as there are G in in the (l.s.c.).

Fact 2. 2 ÷L is I/ (T .V.) part of L .

Fact 2.4 Also ÷L is I/ [(T.V.) − I] PART OF. (column of K).

Fact 2.5 And (÷L ÷ K) ≃ [(l.s. $)^2$ / (T.V.)] PART OF G.

Note: If it should appear to any of my readers that there is a discrep~~ANCY BETWR~~ancy between the number of L calculated from the diagram on page 22.a and the number of L produced by the G of the column of S on page 2I .a ; please remember that all the L of the S in xch are transposed to the position above xch, and all the K in the column of S is transposed to xch.

Laurence M. Rash
Union, Iowa

Section 4I : Analysis of G Space, Abstract proof:

Article 2.

Monday, Feb. 2I, 1944.

I — Diagram I

The d width of (l.s.c.) across the two rows of ch gives us the value xch. When all the ch in (l.s.c.) are placed above the two rows of ch, they can ~~expre~~ be expressed as a column of xch. It is quite proper to add or jointhe column of xch and the xch across the two rows of ch, together in a sum. This sum is less than (l.s.c.) because there is the value L to the right of the column of xch, remaining.

II — Diagram 2.

The width of the column of xch across the two rows of ch gives the value S. $(xch \dotminus S) = K$. Appling this equation to the column of xch, we get 2 columns; the column of S, and the column of K. It is proper to join the column of S, and the S across the two rows of ch together in a sum. The column of S does not equal the column of xch because we have the column of K remaining.

III — Diagram 3.

It is proper to join the column of K, to the K across the two rows of ch in a sum. (These values are of diagram 2 to be transfered to diagram 3). This sum must have an 七 added to equal a G — space. This G — space (diagram 3) is an exact miniature of (l.s.c.) diagram, horizontally only. The heights of the two diagrams are the same. It is (I / T.V.) part of (l.s.c.).

The column of K — 七.

IV

All three diagrams are the same height. The difference between diagram I and diagram 3 is the ratio of their widths. While diagrams I and 3 are exactly alike except for scale; diagram 2 is different because it has two 七 but only one corresponding 七 and one K.

V — (Considering only the values of (l.s.c.) above 2rows of ch .)

Total sum of K = xch.

(T.V.) K ≡ XCH. (diagram Page 22.a)

I) Then (T.V.) columnof K ≡ xch column.
And (T.V.) 七 = L.

Completes (l.s.c.) above 2rows of ch.

~~2) Calculatio~~

(Continued page 24 .)

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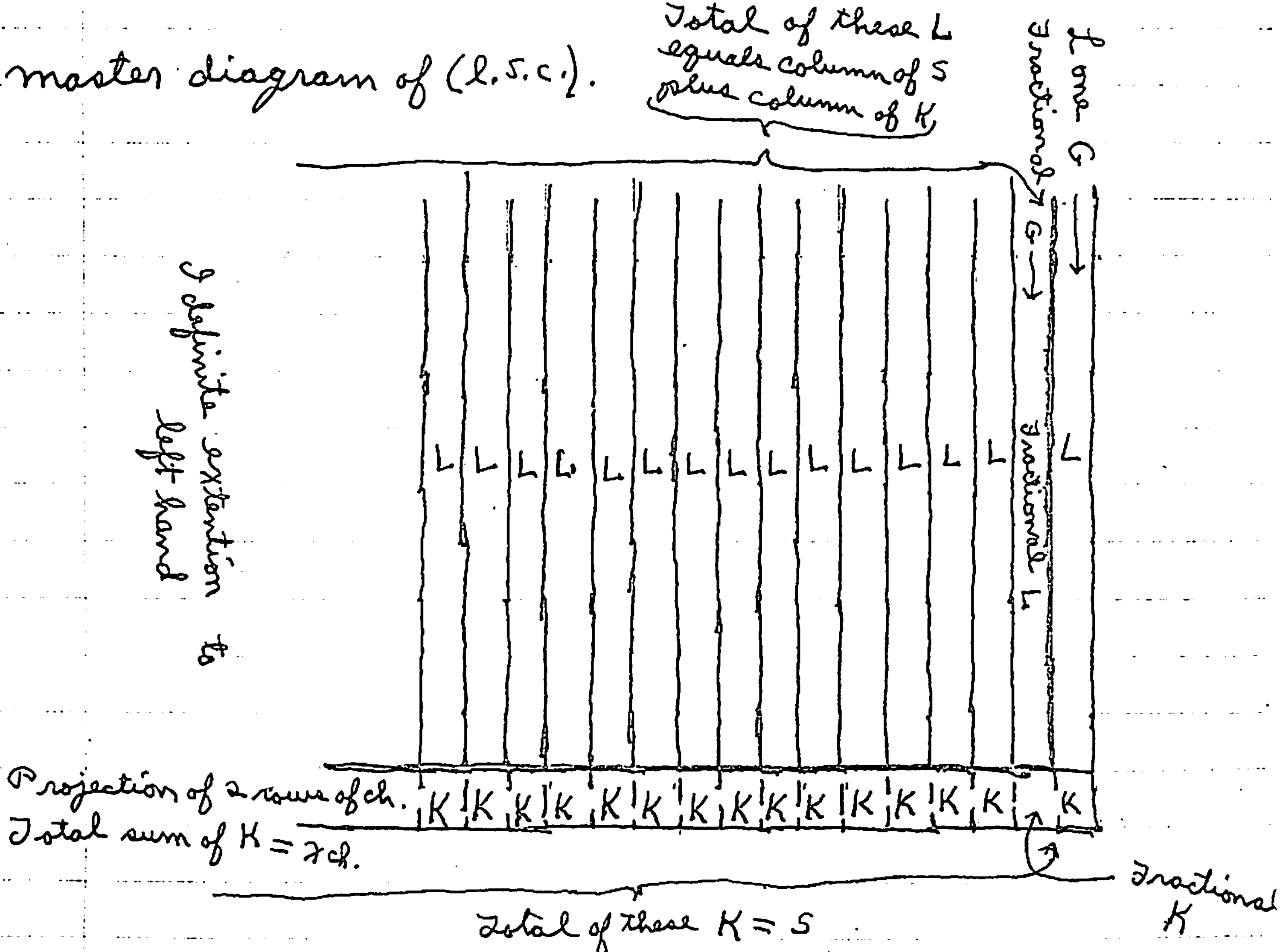
Section 41: analysis of G space
abstract proof:

article 1.

as this diagram proposes to deal with all possible circumstances: the top is left open so that the height may be extended indefinitely, or made less; and the left hand side is left open so that (T.V.) may be given any desired value.

master diagram of (l.s.c.).



In this diagram the column of K is "leveled off" to form a fractional G. This is almost actually done when later I explain the small increased (l. S. c.)

Page 23.a

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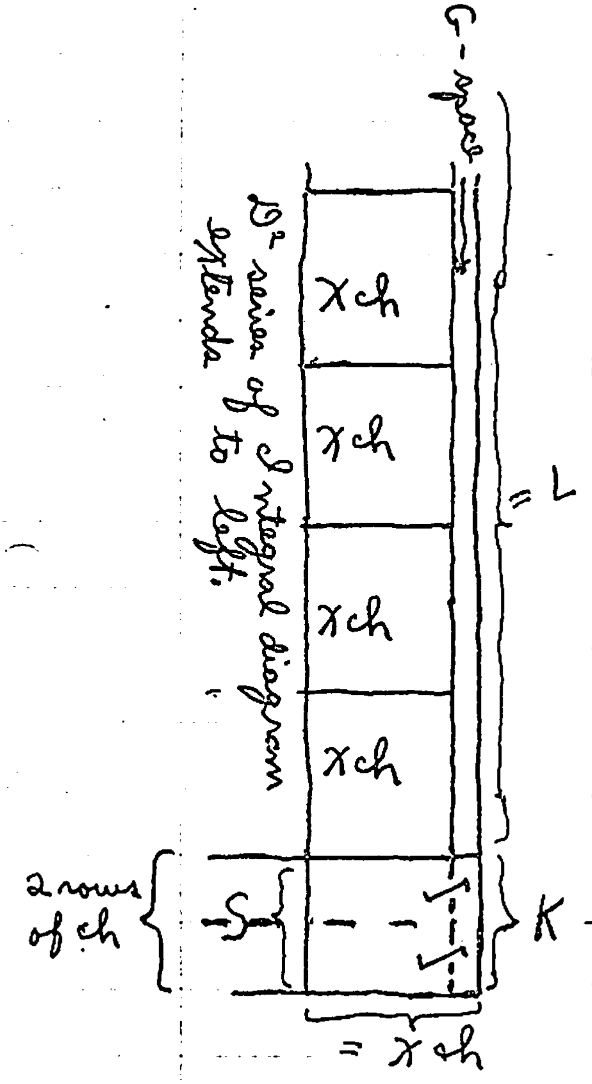
Section 41: analysis of G Space
abstract proof:

article 2.

Monday, Feb. 21, 1944

Diagram of (l.s.c.) showing column of Xch



(S + K) = Xch

Diagram 1

Diagram of (l.s.c.) showing column of S and column of K

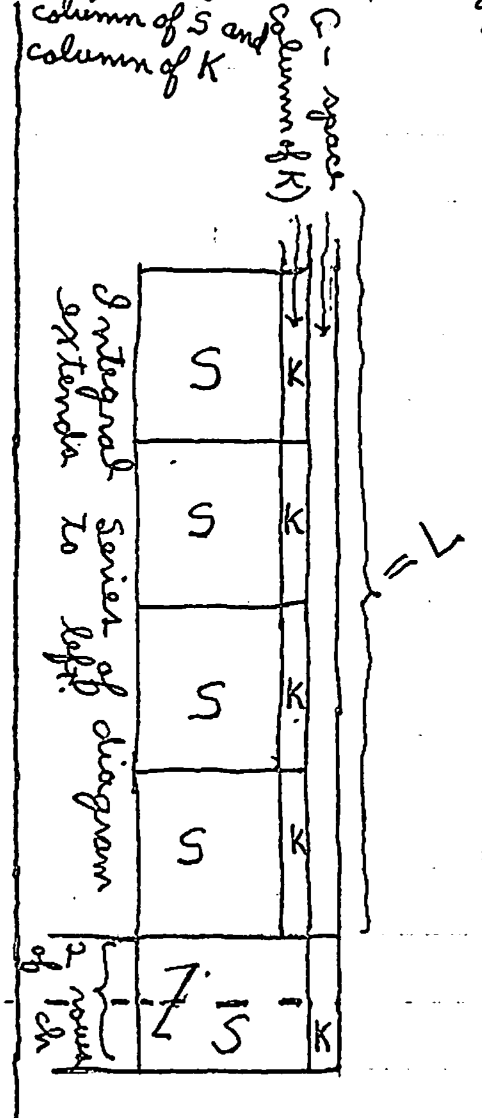
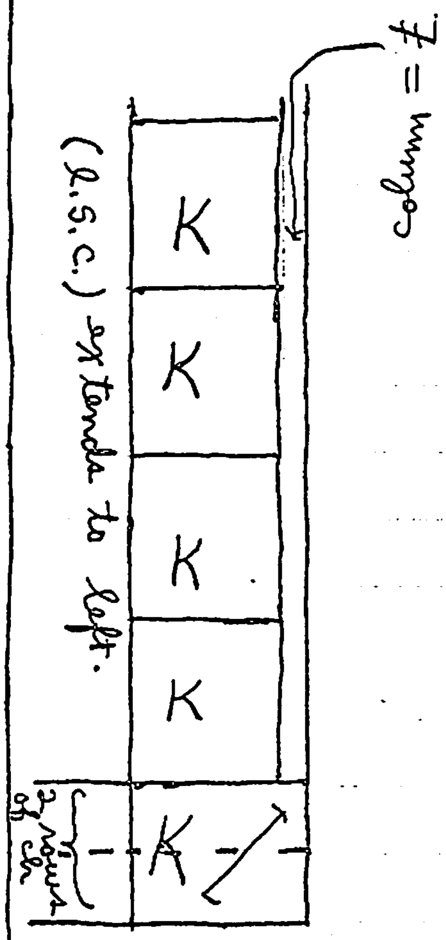


Diagram 2

Diagram of G-space showing column of K and column equal to Ł



(column of K) = Ł

Diagram 3

Section [illegible]I:
Article 2

2) Calculating again:
Then (T.V.) column of K = xch column.
And (T.V.) – I times [illegible] = column of K.
[illegible] = [illegible].

Completes (l.s.c.) above 2 rows of ch.

3) K is contained in S , (T.V.)– I times;
And (column of K) is contained in (column of S),
(T.V.) – I times.

Therefor (T. V.) – I times (column of K) = (column of S).
And (T.V.) – I times [illegible] = (column of K).
[illegible] = [illegible].

Completes (l.s.c.) above the 2 rows of ch.

Note: Whereas the column of K is only a fractional L , of a fractional G, after it is redistributed into a sum, as under item III , Page 23: then if the xch column contains a number of L, whose coefficient is an integer number; the complement of this fractional L is in (column of S).

THERE ARE AS MANY [illegible] IN L , AS THERE ARE G IN (l. s.c.) .

Page 25.
Section 4 2:

Laurence M. Rash
Union, Iowa
March I6, I944.

Numerical proof of values in (l.s.c.) :-

My purpose in this section is to construct a complete diagram, in which the value of N is a known number. N, being a known number, the values of all other quantities can be easily calculated. Having this advantage, I shall be able to furnish numerical proof of relationships whose validity I have not been able to establish as plainly as I would have wished, abstractly. Of course, strictly speaking, the validity Would be established beyond controversity only for this case alone. However, I believe that any mathematician will see that I have a theory that will hold good in a much larger area. Add to this, the abstract proof I have presented under Section 4 I: Articles I and 2.

Here in Section 4 2, under items I and II I calculated the complete and integral diagrams.

Under item III I found the numerical value of (l.s.c.) by subtraction.
Note: On page 3.c, Section 6, I defined "V" as the ordinal number of the differences of squares. Under this item, Item III, page 25.b, I have given a too generalized meaning to "V". In this case, "V" is used to ~~[illegible]~~ number of divisions", respectively in the complete and integral
mean " the diagrams.

Under item IV I have verified my calculations by proving that my values for the parts give a sum equal to the whole complete diagram.

Under item V, I have calculated (l.s.c.) abstractly so as to have a structural theory on which to base numerical calculations.

Under item VI, I found that the total of the numerical values of the parts of (l.s.c.) equaled the numerical value of (l.s.c.).

Section 43 :

I wish to call attention to the width of the column of all ch in (l.s.c.) or in other words, the xch column, page 25.c; item V.(b). Here the width is given as 40 / 8I (l.s.). Another example of width of xch column is found under Section 39, page 20.c In this instance it is 84 / I69 (l.s.). Here are two examples of the Rule under Section 39, Page 20:- Experimentation with a number of diagrams, shows that the coefficient of the width of the x ch column in (l.s.) units, is always a little less than I/2.

Laurence M. Ra
Union, Iow

Section 42:

numerical proof of values in (l.s.c.)

March 16, 1944

My purpose in this section is to constru a complete diagram from known values of a, n, d, l.s., v, etc. Having the advantage of know values, I shall be able to furnish numerical pro of rules whose validity I have not been able to establish as plainly as I would have wished.

I - Construction of complete diagram. I am using this diagram because I have the calculation alread made and proven.

Let $d = 5$, $l.s. = 2$, v of $n = 4$,
$n = 22$, and $a = 27$.

$$\begin{array}{r} 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline 484 \end{array} \qquad \begin{array}{r} 27 \\ 27 \\ \hline 189 \\ 54 \\ \hline 729 \end{array} \qquad 245\overline{)729}(2 \quad \begin{array}{r} 490 \\ \hline 239 \end{array}$$

$n^2 = 484$ $\qquad$ $a^2 = 729$

$$\begin{array}{rl} a^2 &= 729 \\ n^2 &= 484 \\ \hline (a^2-n^2) &= 245 \end{array}$$

Length of complete diagram below $n^2 = 1\frac{239}{245}$ divisions.

II - Calculation of integral diagram.

Below n^2 are $16 d^2$. as (a^2-n^2) is the 5 th difference it equals $9 d^2$.

$16 \div 9 = 1\frac{7}{9}$. Length of integral diagram below $n^2 = 1\frac{7}{9}$ divisions.

Laurence M. Rash
Union, Iowa

age 25. b

ection 42:

Calculating the numerical value of (l.s.c.).

v of complete diagram below $n^2 = 1\frac{239}{245}$ d.

v of integral diagram below $n^2 = 1\frac{7}{9}$ d.

Place these fractions over the common denominator 9(245).

Subtract $\begin{array}{r} 2151 \\ 1715 \\ \hline 436 \end{array}$ $\begin{array}{r} 245 \\ 7 \\ \hline 171.5 \end{array}$ $\begin{array}{r} 239 \\ 9 \\ \hline 2151 \end{array}$

v of complete diagram $= 1\frac{2151}{9(245)}$.

v of integral diagram $= 1\frac{1715}{9(245)}$.

Then $\frac{436}{9(245)}$ d = width of (l.s.c.).

$(l.s.c.) = \frac{436}{9(245)}$ division. $\begin{array}{l} 9)\,436\,(\underline{48} \\ \underline{36} \\ \;76 \\ \;\underline{72} \\ \;\;4 \end{array}$

$\frac{436}{9(\cancel{245})} \times \frac{\cancel{245}}{1} = \frac{436}{9}$

numerical value of (l.s.c.) = $48\frac{4}{9}$.

Verification of these numerical values for complete diagram

From given known values: ch = 10; $d^2 = 25$.

$2 \times 1\frac{7}{9} = 3\frac{5}{9}$. ∴ Then $3\frac{5}{9}$ ch is under integral diagram.

$3\frac{5}{9}$ ch = $35\frac{5}{9}$

(l.s.c.) = $48\frac{4}{9}$

$16d^2 = \underline{400}$

Total = 484

as $n^2 = 484$ this is correct.

Page 25. ba

Laurence M. Rash
Union, Iowa

Section 42:

III – Calculating the numerical value of (l.s.c.).

number of divisions:

of complete diagram below $n^2 = 1\frac{239}{245}$;

of integral diagram below $n^2 = 1\frac{7}{9}$.

as $(a^2 - n^2)$ has 1 d length, the length of these two diagrams may be expressed as $1\frac{239}{245}$ d and $1\frac{7}{9}$ d. Place these fractions over their common denominator. 9(245).

Subtract
$$\begin{array}{r} 2151 \\ 1715 \\ \hline 436 \end{array} \qquad \begin{array}{r} 245 \\ 7 \\ \hline 1715 \end{array} \qquad \begin{array}{r} 239 \\ 9 \\ \hline 2151 \end{array}$$

number of divisions:

of complete diagram below $n^2 = 1\frac{2151}{9(245)}$;

of integral diagram below $n^2 = 1\frac{1715}{9(245)}$.

Then $\frac{436}{9(245)}$ d = width (length) of (l.s.c.).

(l.s.c.) = $\frac{436}{9(245)}$ division.

Return to page 25. b.

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note:

I refer to the direction of addition of divisions of a diagram as the length. also this is the direction of addition of units of the arithmetical Top row of the progression.

I refer to the same direction as the width of (l.s.c.) because its horizontal dimensional is much smaller than its verticle.

Page 25.c

Section 42:

Laurence M. Rash
Union Iowa

V - Calculation of (l.s.c.) abstractly:

The sum of all (l.s.) values below n^2 (in other words, if the l.s. space was placed just below n^2, and above all the 4d differences) would equal $(l.s.)^2 + 8$ ch.

There are $3\frac{5}{9}$ ch beneath the integral diagram. By subtraction, the $(l.s.c.) = (l.s.)^2 + 4\frac{4}{9}$ ch. Call the ch at the bottom of (l.s.c.), across the 2 rows of ch, = $x$ch. width of (l.s.c.) $= \frac{436}{9(245)}$ d.

$$2 \times \frac{436}{9(245)} \text{ ch} = \frac{872}{9(245)} \text{ ch} = x\text{ch}.$$

(a) Rearrange (l.s.c.), placing all ch values above the 2 rows of ch. $4\frac{4}{9}$ ch $= \frac{40}{9}$ ch.

$$\frac{40}{9} \text{ ch} = \frac{9800}{9(245)} \text{ ch}.$$

$$872)\overline{9800}(11$$
$$\underline{872}$$
$$1080$$
$$\underline{872}$$
$$208$$

$$8 / \frac{208}{872} = \frac{26}{109}$$

There are $11\frac{26}{109}$ $x$ch above the two rows of ch in (l.s.c.).

(b) Calculation of S and G.

The height of the diagram above the 2 rows of ch is 9 d.

Place all the ch in (l.s.c.) above the 2 rows of ch. Divide $\frac{40}{9}$ ch by $9 = \frac{40}{81} =$ [illegible] width of column of ch or $x$ch column.

as the height is in d, the width must be in l.s. value. Then width $= \frac{40}{81}$ l.s.

Section 42:

The extention of this column across the 2 rows of ch gives a space equal to $\frac{80}{81}(l.s.)^2 = S$. Then $\frac{1}{81}(l.s.)^2 = G$ space, the remainder.

note: On page 20, under Section 39, we have the statement, "Experimentation with a number of diagrams shows that the coefficient of this (l.s.) width is always a little less than $\frac{1}{2}$." In the above case, the width of $x$ch column $= \frac{40}{81}(l.s.)$

VI – numerical verification of the values of parts of (l.s.c.).

The purpose of this verification is to prove above controversy that the theory of Diagram B^2, page 23.a is entirely correct when applied to this ~~(l.s.c~~.) particular (l.s.c.). The reason I am able to do this, is that n is a known value. Therefor the values d, l.s., ch, etc. are known numerically. I would be unable to know this when working with a surd.

I have established these principles by numerical calculations for this one case. I believe these principles are true in all cases.

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VI – continued:

verification of values inside (l.s.c.).

There are $11\frac{26}{109}$ Xch above the two rows of ch in (l.s.c.). (page 25.c, under item (a)).

Then $(l.s.c.) = 12\frac{26}{109}\ Xch + L.$

a variation of this equation, and the one I shall use in this calculation is:

$$(l.s.c.) = 12\frac{26}{109}\ S + 11\frac{26}{109}\ K + G.$$

$S = 80\,G.$

$$12\frac{26}{109}\ S = \frac{1334}{109}\ S = \frac{106720}{109}\ G.$$

$$\frac{106720}{109}G + 1G = \frac{106829}{109}G\ .$$

numerically $(l.s.) = 2$ and $(l.s.)^2 = 4$.

$G = \frac{1}{81}(l.s.)^2$. numerically $G = \frac{4}{81}$.

$\frac{106829}{109}G = \frac{427316}{(109)\ (81)}$ numerically.

$$\begin{array}{r} 109 \\ 12 \\ \hline 218 \\ 109 \\ \hline 1308 \\ +\ 26 \\ \hline 1334 \\ \times\ 80 \\ \hline 106720 \end{array} \qquad \begin{array}{r} 106829 \\ 4 \\ \hline 427316 \end{array}$$

Height of diagram = $9d + 2(l.s.)$

as $d = 5$, $l.s. = 2$ (page 25.a)

In G space height of $L = 45$; of $K = 4$.

$K = \frac{4}{49}$ of G. $\quad 11\frac{26}{109}K = \frac{1225}{109}K$

$\frac{1225}{109} \times \frac{4}{49} \times \frac{4}{81} = \frac{19600}{(109)(49)(81)}$ numerical value of Ks

$$\frac{427316}{(109)\ (81)} = \frac{20938484}{(109)(49)(81)}$$

$$\frac{20938484}{(109)(49)(81)} + \frac{19600}{(109)(49)(81)} = \frac{20958084}{(109)(49)(81)}$$

numerical value of (l.s.c.).

$$\begin{array}{r} 109 \\ \times\ 11 \\ \hline 109 \\ 109 \\ \hline 1199 \\ +\ 26 \\ \hline 1225 \end{array} \qquad \begin{array}{r} 1225 \\ 16 \\ \hline 7350 \\ 1225 \\ \hline 19600 \end{array}$$

$$\begin{array}{r} 427316 \\ 49 \\ \hline 3845844 \\ 1709264 \\ \hline 20938484 \\ +\ 19600 \\ \hline 20958084 \end{array}$$

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VI – Continued – 3rd page.

numerical value of (l.s.c.) = $\frac{20958084}{(109)(49)(81)}$.

$432621 \overline{)\,20958084}\,(48$

1730484

3653244

3460968

192276

$9 \,\underline{)\,432,621}$
48069

$4 \,\underline{)\,192276}$
48069

$48069 \left| \frac{\cancel{43}192276}{432621} \right. = \frac{4}{9}$

109
81
109
872
8829
49
79461
35316
432621

This, gives the numerical value of (l.s.c.) as $48\frac{4}{9}$.
This is the same as under III, page 25. b.

whereas: I have used the equation and gotten the correct value of (l.s.c.); the equation and diagram 2, page 23. a are proven correct.

Section 44 : Analysis of G S pace

.Abstruct Proof:

Under Section 4 I , Arti-cle I, we have an abstract proof of analysis of (l.s .c .). Under Fact 2.I , Pa ge 22, we have the statement, "T he G Space i s a minature of (l.s.c.)" also "There are as many $\underline{\pm}$ i n ∟ as ~~they~~ there are G in the (l.s.c.) ".

Again under Article 2, Pages 23, 23.a, and 24 we have more abstract proof .

proof.

Now in Section 44 I give two examples of numerical proof of the Rule : "THERE ARE AS MANY $\underline{\pm}$ IN ∟, AS THERE ARE G IN (L.S.C.)."

It is impossible to furnish this numerical proof without calculations from adiagram, the value of whose N is known. The value of N being known all values can be calculated. I have used two such diagrams: one from Page 25. a; Section 39, for Example I; and the other, from Section4 2, Pages 25,a-25.f; for Example 2.

In each example I have calculated the number of G in (l.s.c.) and found them equal to the number of $\underline{\pm}$ in ∟.

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Section 44:

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Example 1

From Page 20.a; Section 39.

Let $d = 5$; v below $n = 6$; and l.s. $= 2$

Then $n = 32$; and $a = 37$.

$n^2 = 1024$; $a^2 = 1369$; and $(a^2 - n^2) = 345$

[Below a^2]; $(2v-1)\,d^2 = 13d^2$; $13d$ = height above 2 rows ch.

as $d = 5$; height above 2 rows of ch $= 65$ = numerical value.

as l.s. $= 2$; height across 2 rows of ch $= 4$ = numerical value.

[all in the lone G.]

Then height of $K = 4$; height of $L = 65$

There are $16\frac{1}{4}$ K in L. [Frist equation]

From page 21.b; Section 40:

There are 16 and $\frac{436}{1784}$ xch in xch column.

There are 16 and $\frac{436}{1784}$ K in Ł. [Second equation]

But there are $16\frac{1}{4}$ K in L.

as $\frac{1}{4}K = \frac{446}{1784}K$:

Then $L - Ł = 16\frac{446}{1784}K - 16\frac{436}{1784} = \frac{10}{1784}K =$ Ⱡ.

16 and $\frac{436}{1784}K = \frac{28980}{1784}K =$ Ł. (Page 21.b)

[Parallel calculation for xch column, page 21.b]

$\frac{28980}{1784}K$ contains $\frac{10}{1784}K$; 2898 times

2898 = number of Ⱡ in Ł.

2898 Ⱡ + 1 Ⱡ = 2899 no. of Ⱡ in L.

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Page 26.b

Section 44 : Example 1 – continued

Page 20.a

$(l.s.) = 2$ and $(l.s.)^2 = 4$ numerical values.

Page 20.c

$\frac{1}{169}(l.s.)^2 = G$ numerical value of G $= \frac{4}{169}$

Page 20.b $\frac{892}{(13)(345)}$ = width of (l.s.c.) in d units.

345 = numerical value of one division.

$\frac{892}{(13)(\cancel{345})} \times \frac{\cancel{345}}{1} = \frac{892}{13}$ = numerical value of (l.s.c.).

Put them over the denominator $(13)(13) = 169$.

$$\begin{array}{r} 4\overline{)11596}\,(2899 \\ \underline{8} \\ 35 \\ \underline{32} \\ 39 \\ \underline{36} \\ 36 \\ \underline{36} \end{array}$$

$$\begin{array}{r} 892 \\ \underline{13} \\ 2676 \\ \underline{892} \\ 11596 \end{array}$$

Thus it is proven that there are 2899 G in (l.s.c.) in this example. This agrees with the number of ± in L.

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Section 44:

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Example 2
From March 16, 1944

Height of complete diagram $= 9d + 2\ l.s.$

Because $(a^2 - n^2)$ is the 5th difference it $= 9d^2 + 2\ ch.$
(Page 25.a; Section 42; Item II.)

$d = 5$; $l.s. = 2$; (Same page, Item I.)

$9d = 45$; $2\ l.s. = 4$

Then $L = \frac{45}{49}$ of G, and $K = \frac{4}{49}$ of G.

and $\text{Ł} = 11\frac{26}{109}K = \frac{1225}{109}K = \frac{4900}{4(109)}K.$

But $K = \frac{4}{49}$ of G. (measured by height.)

Then $K = \frac{4}{45}$ of L, and $L = \frac{45}{4}K = \frac{4905}{(4)(109)}K.$

$$L - \text{Ł} = \frac{4905}{(4)(109)}K - \frac{4900}{(4)(109)}K = \frac{5}{(4)(109)}K = \text{Ł}.$$

$5\,\overline{)4905}$
981

Then $L = 981\ \text{Ł}.$

$48\frac{4}{9} = \frac{436}{9}$ = numerical value of (l.s.c.)
(Page 25.b, Section 42, Item III.)

as $(l.s.) = 2$, $(l.s.)^2 = 4$

and $G = \frac{1}{81}(l.s.)^2$

The numerical value of $G = \frac{4}{81}$.

The common denominator $= (9)(9) = 81.$

$\frac{436}{9} = \frac{3924}{81}$. $4\,\overline{)3924}$
981

$(l.s.c.) = 981\ G.$

There are as many Ł in L, as G in (l.s.c.)

Of course this value is T. V..

Chapter VII
Maturity

Page 27 to Page 42
Section 45 to Section 54

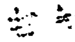
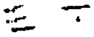
~~Chapter~~
Chapter VII- Maturity.

My effort in this chapter is to find the value of T .V. -in other words the number of G in (1. s.c.). As I explained under Article III, ~~words~~ Section 4 0, Page 2I the possesion of this value is tantamount to a solution of this problem.

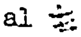
When dealing with surds, (T.V.) is unknown therefor an (Inc. V.) must be used. [From dictionary, "Increase - To become greator, grow, advance, wax. "] For Increased value, I shall use (Inc. V.).

When a (column of K) is altered into afractional G, I call it the " Top G ".

Section 45.

The (column of K) can be made into an entire G by the addition of a K and a lone  by increasing (1.s.c.)the amount of a  Then an approximate value of (T.V.) can be found by counting, or estimating the number of G in (1.s .c.). This solution, I call the ~~(Lrge Increased~~ (Large Increased T .V.)- ---(La-r. Inc. L.S.C.).
Correction: ""by increasing (1.s.c.)the amount of a  K ".

Section 4 6 :

The (column of K) can be made into a sum as under Diagram 2, Item III, Article2, Section 4 I , Page 23. This sum is a fractional (column of K) plusa fractional K - the total equals the entire (column of K). These values are all within (1.s.c.). Now all that is needed to produce a fractional G , whose value in G unites is known; is to add a fractional . The fractional coeffifients of both the added  and the G have the same value, The (1.s.c.)is increased by the value of the fractional . I call this the small Increased (1.s.c.). ---(Sm. Inc. L.S.C.) . This gives a much closer approximate value of (1.s. c.).

Page 27 and I / 2 — Inserted.

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Feb. 8, 1967.

Section 4 6 and I/2.

The reason for inserting this page, is that the reader may be imformed in what part of the following theories to look for progress of the solution of the problem. Thus his interest will be enriched by making compartsions and collations.

Allow me to prequote apart of item V, Page 28 "As we will be dealingwith a (S m. Inc. v.) Top G, the (column of K)of of the Top G can be redistributed into a fractional sum of (column of K s) plus K. But when (T. V.) is unknown, this fractional sum connot be redistributed the second time to get the proper value for the lone (T. V.) 乞, " because the exact coefficient of the (column of K s), expressed in values of G is unknown, when working with surds. This is the original problem of extracting square root on a tiny scale.
The immediate purpose is to calculate the exact number , or a close approximation, of the number of G s in (T.V. 1sc).

This problem is solved, with an infinitely small error, by means of three processes:. The preparatory Two kinds of 乞 S statement (Section 4 7) and The Excess of 乞 Theory (Section 4 8); The Segre-~~ti~~ga ting Diagram (Section 4 9); and a sertes of values which I call Intangibles.
It took me from May 1954, until June 1956 to completely understand " The Excess of 乞 Theory".

Section 47: ------------- Two Kinds of 之 S.

FRom May 5, I954, Tablet II.

There are two kinds of 之 ----the (T. V.) 之 and the (Inc. V.) 之.
~~The (Sm. INc. V.) 之~~

The (Sm. Inc. V.) 之 is the one used.

These two 之 s are exactly the same size, but they are not identical nor interchangeable, because of their grouping within different kinds of 之 S, or K s. The reason is that the (Sm.Inc.V.) 之 S are the units of a (SM. Inc. V.) 之 or K. Their size has not been increased but their number has been. Therefor because of their numerical coefficients, they can not be interchanged with (T.V.) 之 For purposes of establishing comparitive values in G unites. Please remenber that when dealing with surds, the (T.V.) coefficients are unknown quantities.

The following items true:

I-All the 之 in a (T. V.)G are (T. V.) 之,

II - All lone 之 S are (T.V.) 之 S, eventhe lone 之 of a (INC.V.)G.

III -All the 之 S in the (column of K), or K s of a (I nc. v.) G are (I nc. V.) 之.

IV - Therefor the number of 之 S in the (column of K s) of the Top G can not be compared to the number of 之 S in a (Inc. V.)G to determine the coefficient of its G, because it does mot contain a (T.V.) 之, A (T. V.) 之 is never calculated at an (INc. V.) rate.

V - As we will be dealing with a (Sm. Inc. V.) Top G, the (column of K) of the Top G can be redistributed into a fractional sum of (column of K s) plus K. But when (T. V.) is unknown, this fractional sum cannot be redistributed the second time to get the proper value for the lone (T.V.) 之, because the proper coefficient will be slightly different. This is because the Increase is entirely on the 之 and the K beneath the L.

This is the difficulty which stands between us and the calculation of the correct fractional coefficient for the G. The solution of this is tantamount to the solution of the entire problem.

Article I — Statement for entire Section 48.

The purpose of this Section is to prepare the way for the calculation of the number of G s in (T.V. l.s.c.) at the (T.V. Rate), to be proformed in the next chapter. Because (T.V.) is an unknow value, — when working with a surd — we use a known approximate, (SM. Inc. V. l.s .c.). The calculations are based on the Rule: " There are as many s in , as G s in (l. s.c.) ". (See Section 44).
The value sought is (T.V.l.s.c.) at the (T.V. Rate) — in other words (T.V.) number of s in each . By Rate is meant the number of s in each .

The value we are sure of is (Sm. Inc. V. l.s .c.) at the (Sm. INc. V.)
Rate. I nc.

There are two different kinds of enlargements: the excess —Article II, and the increases — Article III.

ARTICLE II — Abstract calculation: Sept. II, I954.

Statement of the Excess of Theory.

The purpose of this theory is to establish a relationship between the abstract values of (T .V. l.s.c.) at the (Sm. Inc. V. Rate) and (Sm. Inc. V. l.s.c.) at the (T. V. Rate).

Because of the disadvantage of working with unknown values, the calculations are somewhat awkward — Extraneous values cannot be separated from the essential values.

These excesses of , are but excesses calculated on the lone . They are no part of the two (l.s.c.) s. But they do show the difference between the values of the two (l.s.c.) s.

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Section 48

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Sept. 11, 1954

Article II – The algebraic Derivation of the Theory.

The (J.V. l.S.c.) at the (Inc. V. Rate Side	The (Inc. V. l.S.c.) at the (J.V. Rate Side

as measured by the number of £ s they contain: —

1. The number of: (J.V.) L at the (Inc. V. Rate = The number of: (Inc. V.) L at the (J.V. Rate).
The numerical coefficient of the £ s for each side is the product of the same two factors.

2. Let x = the number of K s in the column of K s; and y = the number of K s in (column of K s + K), or in other words $y = x+1$.
In order to include the K beneath the L, we multiply by y/x. This ignores the fact that the lone £ is not a part of £. As the factors are the same, the products are equal.

y/x {(J.V.) L at the (Inc. V. Rate)} = y/x{(Inc. V.) L at the (J.V. Rate)}

{(J.V. l.S.c.) at the (Inc. V. Rate) plus excess} = {(Inc. V. l.S.c.) at the (J.V. Rat plus excess

3. Thus for each G, we have $1/x$ £ too many £; we have a complete G + $1/x$ £.

4. For (J.V. l.S.c.) at the (Inc. V. Rate) we have an excess of (J.V.) $1/x$ £. | For (Inc. V. l.S.c.) at the (J.V. Rate) we have an excess of (Inc. V.) $1/x$ £.
Subtract (J.V.) $1/x$ £ from both sides.

(J.V. l.S.c.) at the (Inc. V. Rate) = (Inc. V. l.S.c.) at the (J.V. Rate) plus {(Inc. V.) − (J.V.)} $1/x$ £.

article III

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Section 48
article III – a numerical calculation of the consequential Increase – not excess – of (I.V. l.s.c.) at the (Inc. V. Rate). The basic calculations are those for the diagram found on pages 25.a, 25.b, 25.c, 25.d, 25.e, and 25.f.

Subarticle 1. Statement of contents: May 17, 1966

In this calculation, section 48, I am dealing with a complete diagram based on an (n^2) whose square root (n) is a known number. When the value n is known, all values of such a diagram can be calculated. The purpose of a calculation with known basic values is to discover principles and relationships which may be used when dealing with a surd.

(From March 16, 1944)

Subarticle 2. a numerical "check up" to make sure that my abstract calculations are correct.

$(l.s.c.) = 11\frac{26}{109}\ K + 12\frac{26}{109}\ S + G$ Page 25.e.

Height of diagram $= 9d + 2\,l.s.$

$9d = 45$ $2(l.s.) = 4$ $K = \frac{4}{49}$ of G.

"Checking up" in numerical values:—

as $(l.s.)^2 = 4$; $G = \frac{4}{81}$; $K = \frac{4}{49}\left(\frac{4}{81}\right) = \frac{16}{(49)(81)} = \frac{16}{3969}$.

$11\frac{26}{109}K = \frac{1225}{109}K = \frac{19600}{(109)(3969)}$.

$12\frac{26}{109}S = \frac{1334}{109}S$; $S = \frac{80}{81}(l.s.)^2 = 80G.$

$80G = \frac{80 \times 4}{81} = \frac{320}{81}$ $(49)(81) = 3969.$

$\frac{1334}{109}S$ times $\frac{320}{81} = \frac{(1334)(320)(49)}{(109)(81)\ (49)} = \frac{20,917,120}{(109)(3969)}$.

(The lone) $G = \frac{4}{81} = \frac{(4)(109)(49)}{(109)(3969)} = 21364$

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article III
Subarticle 2 – continued:

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The denominator is
$(109)(3969) = 432621$

$$\begin{array}{r} 3969 \\ 109 \\ \hline 35721 \\ 3969{}_{1}{}^{0} \\ \hline 432621 \end{array}$$

Total of numerators:

$$\begin{array}{r} 19600 \\ 20917120 \\ 21364 \\ \hline 20,958,084 \end{array}$$

$$432621 \overline{)20958084} (48$$

$$\begin{array}{r} 1730484 \\ \hline 3653244 \\ 3460968 \\ \hline 192276 \end{array}$$

$$48069 \Big/ \frac{192276}{432621} = \frac{4}{9}$$

$48\frac{4}{9}$ = numerical value of (l. s. c.) – complete agreement with Page 25. b, Section 42, article III

Therefor my abstract calculations are correct.

Subarticle 3. The ~~numerical value~~ abstract calculation Inc of (S.m.v.-l.s.c.) in values of G. Subarticle 2, just above, gives the abstract value as $12\frac{26}{109}S + G + 11\frac{26}{109}K$. The following abstract calculations can be done even with a surd.

$S = 80\ G$; Then $12\frac{26}{109}S = 979\,{}^{9}\!/\!{}_{109}\,G$.

$$\begin{array}{r} 12 \\ 80 \\ \hline 96{}_{1}{}^{0} \end{array} \quad G \qquad \begin{array}{r} 26 \\ 80 \\ \hline 208{}_{1}{}^{0} \end{array}$$

$$109 \overline{)2080} (19$$

$$\begin{array}{r} 109 \\ \hline 990 \\ 981 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 19\frac{9}{109} \\ \hline 979\ {}^{9}\!/\!{}_{109} \end{array}$$

G – total value of $12\frac{26}{109}S$.

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Section 48
article III
Subarticle 3 – continued:

$11\frac{26}{109}$ K contains $\frac{1225}{1334}$ of the total K in a entire G.

$11\frac{26}{109} K = \frac{1225}{109} K$; $12\frac{26}{109} K = \frac{1334}{109} K$.

Therefor after the addition of $\frac{1225}{1334}$ ± the (column of K s) becomes $\frac{1225}{1334}$ G.

For (Sm. Inc. V. l. s. c.) we have in values of G:

$$979\frac{9}{109}\,G \quad = \quad 979\frac{12006}{(109)(1334)}\,G$$

The lone $1G = 1\ G$

$$\frac{1225}{1334}\,G \quad = \quad \frac{133525}{(109)(1334)}\,G$$

The denominator (109)(1334) $\Big\}$ =

$$\begin{array}{r} 1334 \\ 109 \\ \hline 12006 \\ 13341^{0} \\ \hline 145406 \end{array}$$

Sum of numerato

$$\begin{array}{r} 12006 \\ 133525 \\ \hline 145531 \end{array}$$

$$\begin{array}{rl} 145531 & = \text{sum} \\ 145406 & = 1G \\ \hline 125 & \end{array}$$

Therefor (Sm. Inc. V. l. s. c.) = $981\frac{125}{(109)(1334)}$ G.

Please note that because of the denominator this fraction, $\frac{125}{(109)(1334)}$ is in G values.

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Subarticle 4 — (J.V. l.s.c.) calculated numerically in G, so as to find the numerical value of (J.V.)

$(l.s.) = 2$; $(l.s.)^2 = 4$

$\frac{80}{81}(l.s.)^2 = S$; $\frac{1}{81}(l.s.)^2 = G = \frac{4}{81}$ numerical value.

$48\frac{4}{9}$ = numerical value (J.V. l.s.c.)

$= \frac{436}{9}$ Put over the common denominator 81.

$= \frac{3924}{81} = (J.V.\ l.s.c.)$

$$\begin{array}{r} 436 \\ 9 \\ \hline 3924 \end{array}$$

Divide by the value of G.

$$\begin{array}{l} 4)\,3924\,(981 \\ \underline{36} \\ \;\;32 \\ \;\;\underline{32} \\ \;\;\;\;4 \\ \;\;\;\;\underline{4} \end{array}$$

Therefor (J.V. l.s.c.) = 981 G
and (J.V.) = 981

Subarticle 5 — This is the heart of Section 48 — a numerical calculation of the consequential Increase — not excess — of (J.V. l.s.c.) at the (Sm. Inc. V. Rate).

These calculations to find {(Sm. Inc. V.) − (J.V.)} follow:

Item 1. — added Increase — The number of G s is increased vertically to equal (Sm. Inc. V.) G by adding $\frac{1225}{1334}$ ± to (J.V. l.s.c.) at the (J.V. Rate).

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article III
Subarticle 5 — continued:

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Item 2 — Horizontally:
There are $981 + \frac{125}{145406}$ G in (Sm. Inc. V. l. s. c.).

The number of ± s in each L has also been made equal to the numerical coefficient of (Sm. Inc. V. l.s.c.) in G values. The Increase in each L equals

$$\{(\text{Sm. Inc. V.}) - (\text{I. V.})\} \pm = \frac{125}{14\,5^{4}06} \pm = \frac{125}{(109)(1334)} \pm .$$

$981 = 9(109) = $ I. V.
multiply by the number of G in (I. V. l. s. c.)
— Calculations at G level.

$$\frac{9(\cancel{109})}{1} \times \frac{125}{(\cancel{109})(1334)} \pm = \frac{1125}{1334} \pm .$$

As the increase is actually on the ± s, this must be multiplied by $\frac{1334}{1225}$ so as to include the K s beneath the L s.

$$\frac{(\cancel{1334})}{1225} \times \frac{1125}{(\cancel{1334})} \pm = \frac{1125}{1225} \pm = \frac{45}{49} \pm .$$

— Calculations at K level.
This is the consequential Increase.

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Article III

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Subarticle 6 – Comparision between added Increase and Consequential Increase.

Put all quantities over the common denominator (1225) (1334).

Then $\frac{1125}{1225}\pounds = \frac{(\overset{(1125)}{\cancel{1225}})(1334)}{(1225)(1334)}\pounds = \frac{1{,}500{,}750}{(1225)(1334)}\pounds$

added Increase is:

$$\frac{1225}{1334}\pounds = \frac{(1225)^2}{(1225)(1334)}\pounds = \frac{1{,}500{,}625}{(1225)(1334)}\pounds.$$

The difference is $\frac{125}{(1225)(1334)}\pounds$

Thus (T.V. l.s.c.) at the (Sm. Inc. V. Rate) is $\frac{125}{(1225)(1334)}\pounds$ greater than (Sm. Inc. V. l.s.c.) at the (T.V. Rate).

Multiplications to place values over common denominator

$$\begin{array}{r} 1334 \\ \underline{1125} \\ 6670 \\ 2668 \\ 1334 \\ \underline{1334} \\ 1{,}500{,}750 \end{array} \qquad \begin{array}{r} 1225 \\ \underline{1225} \\ 6125 \\ 2450 \\ 2450 \\ \underline{1225} \\ 1{,}500{,}625 \end{array}$$

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article III

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Subarticle 7— agreement of Subarticle 6 with Excess of Ⱡ Theory.

The Excess of Ⱡ Theory concludes:
(J. V. l. S.c.) at the (Sm. Inc. V. Rate) = (~~Sm~~ Sm. Inc. V. l.S.c.) at the (J. V. Rate) plus {(Sm. Inc. V.) − (J. V.)} $\frac{1}{x}$ Ⱡ.

In this diagram:

(Sm. Inc. V.) = 981 $\frac{125}{(109)(1334)}$ (Subarticle 3)
(J. V.) = 981 (Subarticle 4)

$$\{(\text{Sm. Inc. V.}) - (\text{J. V.})\} = \frac{125}{(109)(1334)}$$

$$\frac{1}{x} = \frac{109}{(1225)}$$

$$\frac{125}{\cancel{(109)}(1334)} \times \frac{\cancel{(109)}}{1225} = \frac{125}{(1334)(1225)}$$ the required ~~coe~~ coefficient of Ⱡ. a perfect agreement.

Subarticle 8— The source of this increase identified.

It is explained under Subarticle 5 that the horizontal increase is actually on the Ł s, instead of on the L s. an Ł = x(K); therefor $\frac{1}{x}$ of the increase on a Ⱡ, would be the increase on a K.

Section 49 — The Segregating Diagram.

July I 8, I 958.

The reason I call this the Segregating DIAGRAM is that, when used with the processes developed in the next chapter, it becomes a powerful mathematical instrument able to separate the (l.s.c.) into its various parts. The values of Remainder of (T.V.l.s.c.) and W can never be absolutely separated, but the values of the derivatives of W can be made to approach zero as a limit and thus eliminate it from the calculations.

The purpose of this diagram is to obtain the numerical value of (T. V.). As I have explained, under Article II I , Section 40, Page 2I , the possession of this value is tantamount to the solution of this problem.

The shape of this diagram is a rectangle, but the relations of its various parts are such that it may be dealth with similarly as with a square.

AS it is so intimately connected with both chapters I debated which chapter to include it in. I have included it in this chapter because it is a summation of the ideas and theories explained in this chapter. ~~chapter~~

This diagram which I have drawn, is not made to any special scale, but is a general illustration, applicable to any (l.s.c.).

Discussion :

This diagram shows (T.V.l.s.c.) at the (T.V. RAte) and its various enlargements.. The horizontal added fractional $\frac{1}{}$ at the top of the diagram is the added increase which enlarges (T.V. l.s.c.) into (Sm. Inc. V. l.s.c.). The W to the left of this added increase is the consequential increase thereon, because of the (Sm. Inc. V. Rate). The verticle equal added fractional [illegible] is the consequential increase on the Remainder of (T.V. l.s.c.). Also there is a consequential Increase on the (I/y) G at the bottom of the page. (See Section 4 8; Article III; Page 30. g; Subarticle 8).

Even though this diagram minus the $\frac{1}{2}$ (I/y)G, is a rectangle; its parts can be expressed in terms of $(c + b)^2$. In which Remainding (T. V. l.s.c.) equals C^2 ; the two increases equal 2cb; and W equals b^2.

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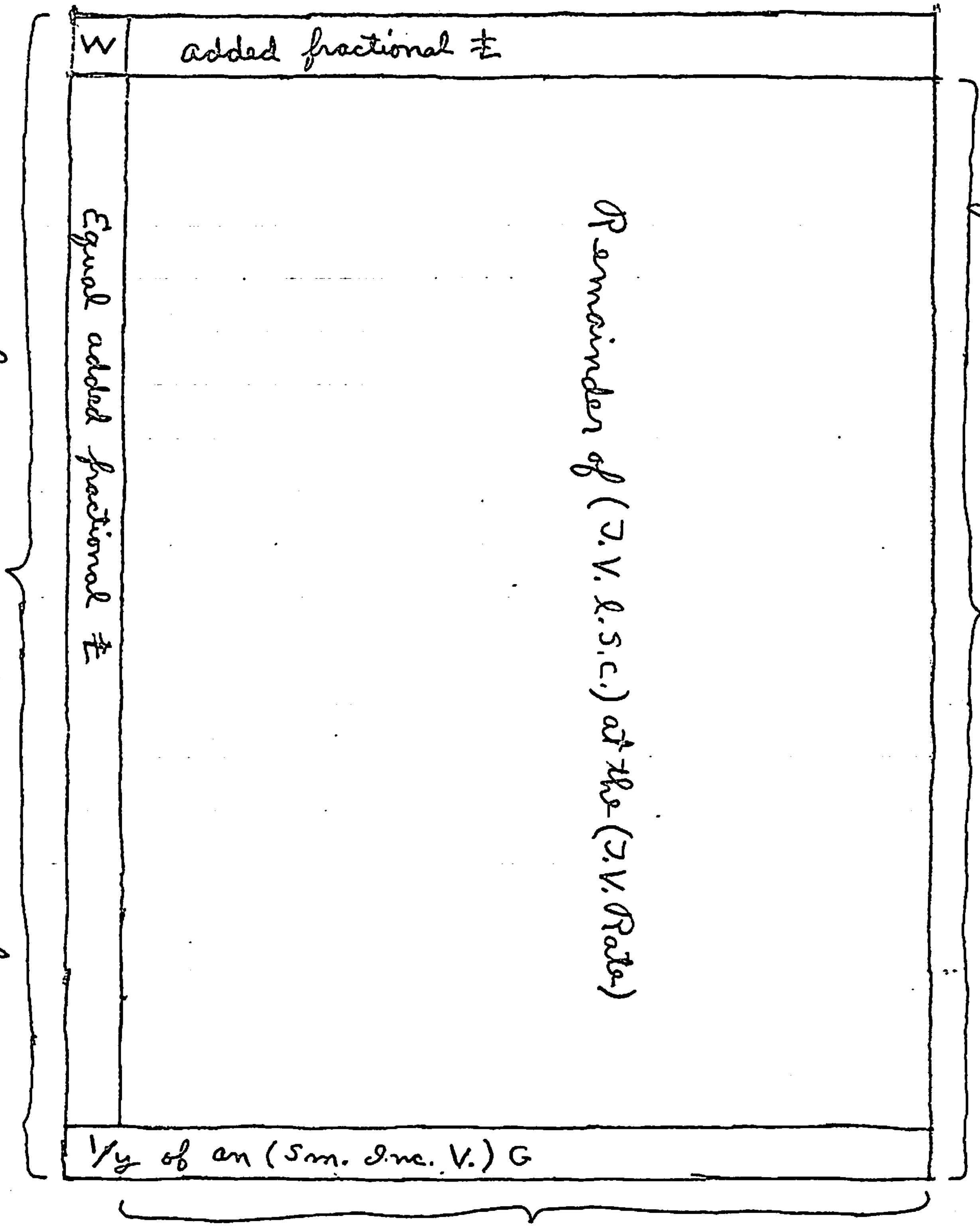
Section 49 – The Segregating Diagram – July 18, 1958

This diagram is a representation of entire (Sm. Inc. V. l.S.C.) at the (Sm. Inc. V. Rate).



(Sm. Inc. V. l.S.c.) at the (S.m. Inc. V. Rate) extends entirely across this diagram = entire diagram.

(J.V. l.S.c.) at the (S.m. Inc. V. Rate) extends across diagram = this diagram minus (added fractional Ƶ + W)

(Sm. Inc. V. l.S.c.) at the (J.V. Rate) extends the entire height of this diagram = diagram minus {W + added fractional Ƶ + Increase on bottom 1/y of an (Sm. Inc. V.) G}.

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VII. B

Chapter V. B - Maturity contioned.

Section 50. Statement; On page 30.g, Section 48, Article III, Subarticle 8, we have the statement, " an ÷ = x (k), therefor I/x of the increase on a ÷, would be the increase on a K ." This was discovered Aug. 27, 1957.

The increase on a K, is the amount the Consequential Increase is greater than the Added Increase. (Page 30.g, Section 48, Article III, Subarticle 8.)

We eliminate from our calculations this troublesome irregularity by subtracting I / y of an (sm. Inc. V.) G at the (Sm. Inc. V.) Rate from the bottom of the Segregating Diagram. What is left equals Remainder of (Sm. Inc. V. l. s.c.) at the (Sm. Inc. V.) Rate.

In this chapter we are working with Remainder of (Sm. Inc. V. L.S. C.) at the (Sm. Inc. V.) Rate, and the parts which compose it. To help the reader visualize the shape of the surface we dealing with, I have found by calculations that "The Remainder of (T.V. L .S. C.) at the (T.V.) Rate is not a square, but a rectangle. Its height is not exactly, but approximately, two tiens times the reciprocal of its width."

Section 5I : The Preparatory Z Theory. (Aug. I , I 958).

Divide the Remainder of (T.V.L.S.C.)by the coefficient of the added fractional ÷ (i.e. Added Increase). Call the quotient Z .

If the Remainder of (T.V.L.S.C.) is divided by the consequencial (Increase)a fractional ÷, above the I /y G, the quotient is also Z , because the divisors are equal.

Therefor in terms of added fractional ÷ : -

Added (Increase) Fractional ÷ = I
Consequential Increase (the same fractional ÷) = I
Remainder of (T.V.L.S.C.)at the (T.V.) Rate = Z .
W = I / Z .

This value for W will be proven graphically in Section 52.
Multipling thru by Z , thus making W equal to one, the sum is :

$$Z^2 + 2Z + I.$$

This is the (Sm. Inc.L.S.C.)at the (Sm. Inc.) Ra-te in values of W .

Remainder of

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Section 52. Intangibles — February to April 1959

Intangibles are a series of values for {Remainder of (T.V.L.S.C.) at the (T.V.) Rate} plus W , or one of its derivitives , which approach (REMAINder of T.V.L.S.C.)at the (T.V. RATE)as a limit. {Remainder of (T.V.L.S.C.) AT THE (T.V.) Rate} can never be separated from the derivitives of W.

These calculaticns are done with Segregating Diagrams.
Frist: The number of ± ² in (Sm. Inc. V.L.S.C.) at the (Sm. Inc. V.) Rate is calculated.
Second: Subtract the (I / y) G at the bottom of the diagra m, in values of ±.

What is left equals Remainder of (Sm. Inc.V.L.S.C.) a t the (Sm. Inc. V.) Rate.

For each Intangible there are two applications of the Segregating Diagram. The first is Recognition of the values of the parts of the Segregating Diagram for that Intangible; the second, is Transformation to the values of the next higher Intangible. These Intangibles will be known numbers, even when dealing with surd s .

The Recognition Segregating Diagram will show the equivalent of the Added Increase, in values of ± ²; as a verticle column in (Remainder of (Sm. Inc. V. L.S.C.) at the (Sm. Inc.V.) RATE, as the Consequential Increase;

Subtract the two equal values, (the Added Increase) and the (Consequential Increase). What is left equals (Remainder of (T.V.L.S.C.)at the (T.V.) Rate plus W. This is the ~~first=Intangible-value.~~ surface from which the first Intangible value is calculated.

continuation from page 37

Set I :

Divide the coefficient of the number of ± ² in the first Intangible surface by the ± coefficient of the Added Increase.

This is the first Intangible value. This is the number of unitsof a quite definite value, but as the first term in a series I call it an Intangible.

The graphic proof I promised in Section 50 :
The number of (Added fractional) ± ² in (Remainder of T.V.L.S.C.)at the (T.V.) Rate is taken as Z . In the Recognition Diagram I have illustrated this by a number of verticle columns each equal to the Added Increase. The blank space to the left of (T.V.L.S.C.) indicates that there may be an unknown number of additional verticle columns. Above each boundary of the columns, I have placed a dot in the Inc. added space (Increase). Thus (Increase added space) is also divided into Z parts. Also the W space above the Consequential Increase is also equal to I/Z (Added fractional ±). In a Transformation Diagram which follows this Recognition Diagram, (T . V.L.S.C.)at the (T.V.) Rate is also divided into Z equal horizontal spaces. Therefor the Consequential Increa se is divided by dots on the extended boundaries into Z spaces, each equal to W_I . This is the Transformation Diagram which has smaller values for the Added Increase, The Consequential Increase, and the dirivitive of W ,

It is evident that the Recognition Diagram, in terms of W equals

$$(Z^2 + 2Z + I) W .$$

Page 33. a
Section 52

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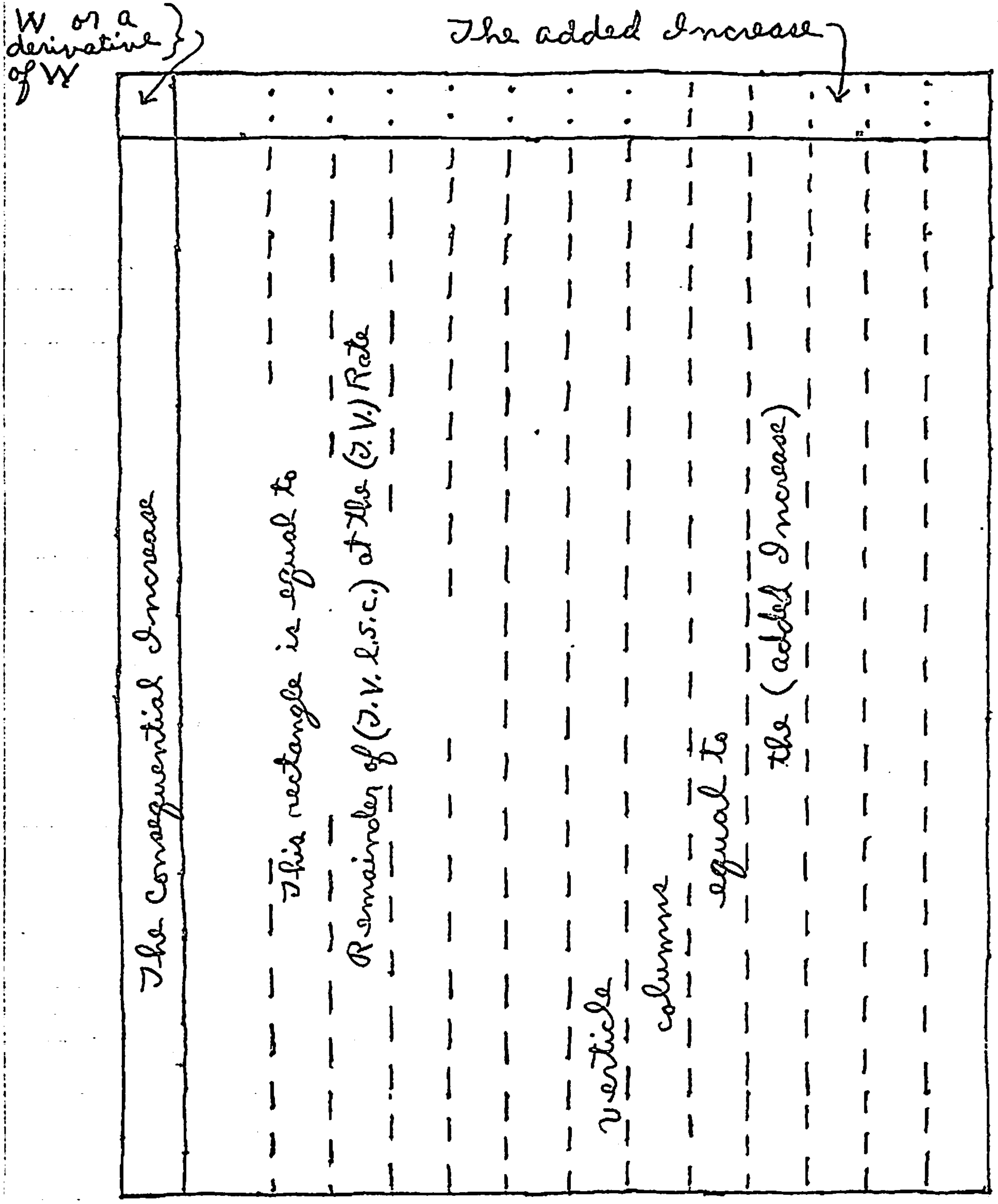
The Recognition Segregation Diagram.

This is a general diagram to show my idea. It may be applied once to each set of values, during any calculation.



This entire Diagram is equal to the Remainder of (Inc. V. l.s.c.) at the (Inc. V.) Rate.

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Section 5 2 – Continued –

After studying the last paragraph on page 53, Ihave decided that I will have my readers confused if I do not rewrite the latter part of it. It says in part "In the Transformation Diagram which follows this (the first) Recognition Diagram, (T.V.L.S.C .)at the (T.V.) Rate, is also divided into Z equal horizontal spaces. Therefor the Consequential Increase is divided by dots on the extended boundaries into Z equal spaces". Also " It is evident that the Recognition Diagram, in terms of W, equals ($Z^2 + 2Z + I$) W ". All this is true.

The Recognition Diagram in terms of the (Added Increase) = (Z + 2 + I/ Z). See Page 32, Section 5 I. Multipling by Z, thus making W equal to one, the sum is ($Z^2 + 2Z + I$). This is collateral proof of the evidence of the Transformation Diagram of the same Set, that Remainder of (T.V.L.S.C.) at the (T.V.)Rate = Z^2 (W). But aside from that, this reasoning leads the theory down a dead end road, from which no progress is possible. The reason is that the theory is "captive" to the conditions and limitations of the Recognition Diagram.

Beginning of the solution.

Set I . The Recognition Diagram , interms of the (Added Increase) equals (Z + 2 + I/Z). Subtract the 2(Added Fractional [illegible]). Then (Z + I /Z) is the Intangible coefficient in terms of (Added Increase) . And W is the derivative of Added Increase.

The Transformation Diagram – From the graphic proof, this Section: " The number of (Added Fractional) [illegible] in (Remainder of T.V.L.S.C.) at the (T. V.) Rate is taken as Z ." These (Added Increases) may be illustrated as Z verticle columns. As we are dealing with the same values, they may also be illustrated as Z horizontal spaces. In the Transformation Diagram both sets of columns or spaces are considered at the same time. Thus (Remainder of T.V.L.S.C.)at the (T.V.) RATE equals Z^2 number of rectangles, which are the intersections of these two sets of (Increases) .

But the W space above the Consequential Increase is also equal to I /Z (added Increase) . Therefor Z^2-(W) W is equal to one of these intersections (rectangles) . Therefor Z^2 (W) EQUALS (Remainder of T. V.L.S.C.)at the (T. V.) Rate; because Z , the number of W in the (Consequential Increase), is taken as its height; and Z , the number of W in (added Increase), is taken as its length.

But Z is an unknown quantity. However the Intangible value from the first Recognition Diagram, (Z + I /Z) is a close approximate (in terms of Added Increase). This is a known number,even with surds. Substitite (Z + I/Z) for Z .

In terms of the derivative of (Added Increase), which is W, this calculation gives:

(Remainder of T.V.L.S.C.)at the (T.V.) Rate = Z^2 (W),
(I / Z) times Z = one W, for each of the Increases,
The derivative of W = (~~I/Z^2~~ I /Z^2) W.
The Intangible coefficient = ($Z^2 + I/Z^2$) .

Set II. The Recognition Diagram:

The values of the Transformation Diagram of Set I are used to calculate the Recognition Diagram of Set II.

NOTE: The values of each Set are in terms of the derivative of W, of the Set just preceeding.

The Intangible coeffifient of Set I times the derivative of W of Set I = THE VALUE OF EITHER THE VERTICLE COLUMNS OR HORIZONTAL SPACES OF SET II .

The height = ($Z^2 + I/Z^2$) (I /Z^2) W

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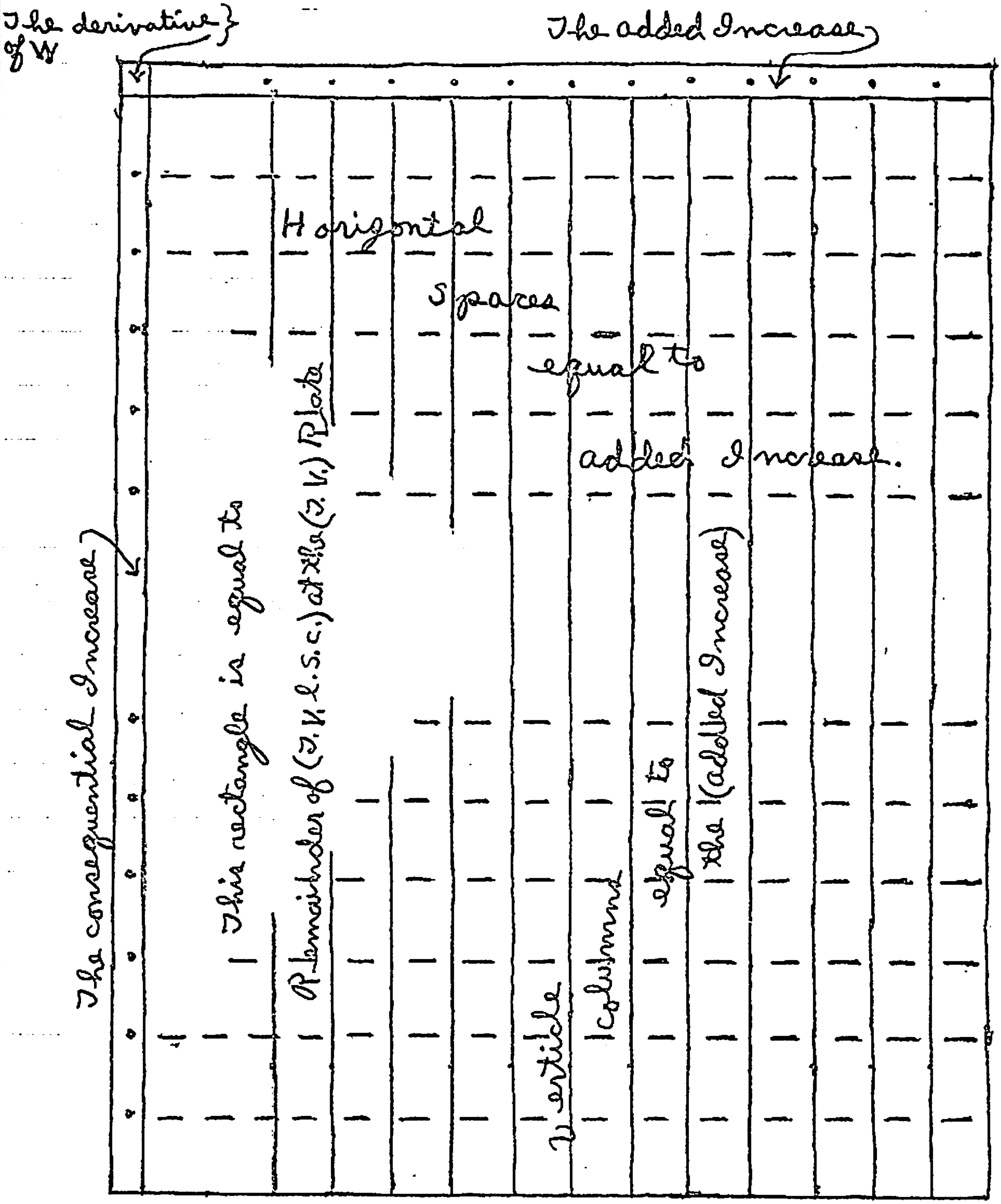
Section 52

The Transformation Segregating Diagram.

The purpose of this Diagram is to complete the developement of the ideas introduced by the corresponding Recognition Diagram.

Notice: Both Increases are smaller.



Note: The (added Increase) inside Remainder of (T.V. l.S.C.) at the (T.V.) Rate is equal to that received from the preceding Diagram. The Increases on the outside are smaller.

This entire Diagram is equal to a reduced Remainder of the (Inc. V. l.S.C.) at the (Inc. V.) Rate.

Section 52 —Continued.

Because of the close continuity of thought, I have allowed the sequence of ideas to proceed until I have confused the Transformation Diagram of Set I with the Recognition Diagram of Set II .. While the calculations are correct , the ideas will become so confused that I can not carry my explanation much further , if I do not establish an orderly method of proceedure. Therefor beginning with the Transformation Diagram of Set I :

We take the value (Z + I/Z) as both the height and width of this Transformation Diagram. The indicating of these values is the completion of Set I .

Set II . The Recognition Diagram is the calculating of the values indicated by the Transformation Diagram just preceeding. To clarify my conceiption of the two kinds of diagrams, I will state that in the Set II, Recognition Diagram; the {Remainder of (T.V.L.S.C.)at the (T.V.) Rate} consists of Z number verticle columns; each of which columns contains Z number of W.
In terms of the derivative of (Added Increase), which is W, this calculation gives:

(Remainder of T .V.L.S.C.) at the (T.V.) Rate = Z^2 (W),
(I / Z) times Z = one W for each of the Increases,
The derivative of W = (I /Z^2) W.
The Intangible Coefficient = ($Z^2 + I / Z^2$) .

The Transformation Diagram:
The height and width are both taken as ($Z^2 + I/Z^2$).
This is the completion of Set II.

~~The Recognition Diagram:~~

Set III . The Recognition Diagram:
The values of the Transformation Diagram of Set II are used to calculate the Recognition Diagram of Set III . For this Recognition Diagram:
{Remainder of (T. V.L.S.C.) at the (T.V.) Rate} consists of Z^2 verticle columns, each of which consists of Z^2 (I /Z^2 W).
In terms of the derivative of W :

{Remainder of (T.V.L. S .C.) at the (T.V.) Rate} = Z^4 (I/Z^2W) ,
Z^2 times (I/ Z^2) = one (I / Z^2) W for each of the Increases,
The derivative of W = (I/Z^4) W .
The Intangible Coefficient = ($Z^4 + I/Z^4$).

The Transformation Diagram : The height and the width are both taken as ($Z^4 + I/Z^4$).

Set IV. The Recognition Diagram:
In terms of the derivative of W , which is (I /Z^4) W:—

{Remainder of (T.V.L.S.C .)at the (T.V.) Rate} = Z^8 (I / Z^4 W),
Z^4 times I/Z^4 = ONE (I /Z^4)W for each Increase.
Derivative of W = (I / Z^8) W .
The Intangible Coefficient = ($Z^8 + I / Z^8$).

V. It is evident that theoretically this process could continue forever . But suppose we decide that (I / Z^8) W is a quantity so small we can neglect it. Then we take ($Z^8 + I/Z^8$) (I/Z^4 W)as equal to {Remainder of (T.V.L.S. C.) at the (T. V.) Rate}.

This is the "End Point" of the advance of the calculations into minute values. From this value for {Remainder of (T.V.L.S.C.)at the (T.V.) Rate}we find its approximate value in fractional $\frac{\ }{\ }$, which is a close approximate of Z .

Section 53: To find the numerical value of Z for the diagram under March 16, 1944.

Statement:

This is a diagram in which n is a known number. We can proform the mathematical operations in the same manner as if we were dealing with unknown quantities and still have the advantage of being able to check for accuracy both, the abstract theory, and the numerical calculations, for the one example alone.

Article I – The preliminary, basic, calculations up to the calculation of (l.s.c.) abstractly – Pages 25.a to Page 25.c, Section 42 – V, the introductory statement only.

Article II – Rearrangement of (l.s.c.) – Page 25.c, Section 42, V (a); calculation of S and G – V (b) – continued Page 25.d.

Article III – Calculation of (S.m.Inc.V. l.s.c.) in values of G. x and y defined Page 29.a.

$$y = x + 1.$$

From Page 25.c – $\frac{872}{9(245)}$ ch $= x$ch.

There are $11\frac{26}{109}$ $x$ch above the two rows of ch, ~~in (l.s.c.)~~. in (l.s.c.). This value $= x$.

Therefor $x = \frac{1225}{109}$. Therefor $y = \frac{1334}{109}$.

$$\begin{array}{r}109\\ 11\\ \hline 109\\ 109\\ 26\\ \hline 1225\end{array} \qquad \begin{array}{r}109\\ 12\\ \hline 218\\ 109\\ 26\\ \hline 1334\end{array}$$

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Section 53:
article III – continued:

Note: one Xch = one S + one K. Therefor the number of K s in the column of K s, is the same as the number of Xch above the 2 rows of ch. — Page 23.a.

The Added Increase = $\frac{x}{y}$ ŧ = $\frac{1225}{1334}$ ŧ — Page 30.c.

The (Sm. Inc. V. l. s.c.) is calculated as equal to $981\frac{125}{145406}$ G — Page 30.c, also 30. e.

145,406 = (109)(1334).

$$981\frac{125}{(109)(1334)}\,G = \frac{142,643,411}{(109)(1334)}\,G.$$

= (Sm. Inc. V. l. s.c.).

$$\begin{array}{r}
145,406 \\
981 \\
\hline
145406 \\
1163248 \\
1308654 \\
\hline
142,643,286 \\
+ \quad 125 \\
\hline
142,643,411 \\
- \quad 145,406 \\
\hline
142,498,005
\end{array}$$

article IV – Calculation of one G
in values of ŧ.

There are also that many ŧ
in each (Sm. Inc. V.) L.
Subtract the lone ŧ.

There are $\frac{142,498,005}{(109)(1334)}$ ŧ in each (Sm. Inc. V.) t.

Multiply by y/x to include the K beneath, in the 2 rows of ch. (See pages 23 and 23.a, Diagram 3).

$$\frac{\cancel{1,334}}{1225} \times \frac{142,498,005}{(109)\,(\cancel{1334})}\text{ ŧ} = \frac{142,498,005}{(1225)(109)}\text{ ŧ} = t + K.$$

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Section 53:

article IV - continued:

add the lone ±.

1225
109
11025
000
1225
133525
+ 142,498,005
142,631,530

$$\frac{142,631,530}{(1225)(109)} \pm = \text{one (Sm. Inc. V.) G at the (Sm. Inc. V.) Rate.}$$

article V - calculating the number of ± s in Remaining (Sm. Inc. V. l.s.c.) at the (Sm. Inc. V.) Rate.

There are $\frac{142,643,411}{(109)(1334)}$ G.

minus $\frac{1}{y}$ G $= \frac{142,631,530}{(109)(1334)}$ G

= Remaining (Sm. Inc. V. l.s.c.).

$\frac{1}{y} = \frac{109}{1334}$

109
109
981
000
109
$(109)^2 =$ 11881

142,643,411
− 11881
142,631,530

note: The numeratora are the same.

$$\frac{142,631,530}{(109)(1334)} \times \frac{142,631,530}{(1225)(109)} \pm$$

= number of ± s in Remaining (Sm. Inc. V. l.s.c.) at the (Sm. Inc. V.) Rate.

142,631,530
142,631,530
0000
427894590
713157650
142631530
427894590
855789180
285263060
570526120
142631530
20,343,753,350,140,900
1122143 3 2 331 1 - carry over.

Denominator Common Denominator equals $(1225)(1334)(109)^2$.

$$\frac{1225}{1334} \pm = \frac{(1225)^2 (109)^2}{(1225)(1334)(109)^2} \pm = \text{added Increase.}$$

Page 36.c

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Section 53:

Article V – Continued – Placing the two values over their common Denominator:

1225
1225
6125
2450
2450
1225
1,500,625

$(109)^2 = 11881$

1,500,625
11 881
1500625
12005000
12005000
1500625
1500625
17,828,925,625

17,828,925,625 is the numerator of Added Increase.
20,343,753,350,140,900 is the numerator of Remaining (Sm. Inc. V. l.s.c.) at the (Sm. Inc. V.) Rate.

To {(Z + 1/2) dividend} division, page 37. b.
{adjective phrase}

Insertion — Regard to Page 33, Section 52;

The paragraph befor, and the paragraph after the title " Set^{3}I" are correct as an abstract theory, but that method would be a clumsy arithmetical proceedure. The easier way is;

I. Calculate one (Sm. Inc. V.) G in terms of ±.
— (Section 53, Article I V; Pages 36.a and 36. b).

2. From (Sm. Inc. V. L.S.C.)in terms of G subtract I/Y (G). What is left = Remainding (Sm. Inc. V. L.S.C.). —Section 52, Article V, Page 36.b .

3. Multiply the coefficient of the number of ±s in one G by the coefficient of the number of G s in Remainding (Sm. Incl V.L.S.C.). The product equals the coefficient of the number of ± s in Remainding (Sm. Inc. V.L.S.G) at the (SM.Inc. V.) Rate. —(Section 53, Article V, Page 36.b.)

4. Divide the coefficient of the number of ± s in the {Remainder of (Sm. Inc. V.L.S.C.) at the (Sm. Inc. v.) Rate} by the coefficient of the number of ± s in the Added Increase. The quotient = the numerical coefficient of (Z + 2 + I /Z). Then merely subtract 2. The remainder = the numerical coefficient of (Z + I/ Z), the Intangible.

Section 53 — continued. Article VI. Late april and early may 1967.

The Intangible: Formula is (Z + I/Z).

Statement: At the top of Page 33, Section 52, I made this statement, "{Remaindinder of (T.V.L.S.C.) at the (T?V?) Rate} can never be separated from the derivative of W ". Intangibles will never completely separate them, but only approach the value of Remainder of (T.V.L.S.C.)at the (T.V.) Rate as a limit. I have now discovered there is another variation of the calcution which will separate them in certain cases — Thatis separate the value of Z from the value of I/Z. An apparent vacant space between the values of Z and I /Z is something I have experimented with, but I did not think it worthy of a place in this discussion until now.

It is only reasonable to suspect that after a long series of zeros in a decimal that we have passed from one value to the other. I call such a series of zeros a "break". These "breaks" are unusual. The usual difficulty is that after the series of zeros, the decimals digits will become a sum of the latter part of Z and the first part of I/Z . This almost makes a separation of the two impossible.

So we proceed to the division as outlined above:
The coefficient of the number of ± s in Added Increase = divisor.
The coefficient of the number of ± s in the {Remainder of (Sm.Inc. V.L.S.C.) at the (Sm. Inc. V.) Rate} = the dividend.
The quotient = the numerical coefficient of (Z + 2 + I/ Z).
From this value subtract 2.
The Remaindet — (Z + I/Z) = first Intangible value.
It should be evident that when we deal with (Z + 2 + I/Z), we are also dealing with (Z + I/ Z).

In this example there appears in the third decimal place in the quotient, the first zero of a series of zeros. This irregularity I call a "Break".

diFosienseesonpphsthewpagebecome ~~clear later, I will try to present two eralle l~~

For a reason which will become clear later, I will try to present two parallel divisions on opposite pages.

Laurence M. Rash
Union, Iowa

May 1967

Statement; Whereas the {(I/Z) dividend} Division is subsidiary to the {(Z + I/Z)dividend} Division; Page 37. b should be presented befor Page 37.ba Also Page 37. d should preceed Page 37.c. Because of a mental quirk concerning the advantage of the right hand page presentation of text over the left hand, I have gotten my page numbers in confusion.

Section 53 :
Article VIII – The { (I /Z) dividend} Division.-

The point at which the series of zeros begins in the quotient of the {(Z + I / Z) dividend } Division is marked on Page 3 7. b. Now if we hypothesize that this point in the quotient is the end of the numerical value of Z , then the balance of the quotient equals I /Z . -

Comparison I :

Because of the long series of zeros it would seem that we have almost separated I / Z from Z .
At this stage in my calculations , I took the quotient as II 4 I05 3. 24 00009– ← Break
If we take II4I053. 24 ≅ Z , as an estamate; then I / II4 I 053. 24 would equal I / Z . .

So as to compare this value with the decimal fraction below the "break" in the quotient of the { (Z + I/ Z) dividend} Division; we change it to a decimal fraction.

1141053.24) 1.00,00,00,00000 (.0000009— ← Break
10269481.16

note:

A close agreement. But still room for a small difference. The last digit is indefinite. I thought, " This result is very near correct. I would have to extend my series of Intangibles thru a large amount of calculations to better it. "

Following comparisions:

As I extended the quotients of both divisions, the agreement continued . Finally I came to the conclusion that this agreement was so fundamental that it would continue thru an infinite number of decimal places. However that was not absolute proof. -

note: The position of the "Break" is located in the quotient of the {(Z + 1/Z) dividend} Division — Page 37. b.

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Union Iowa
Fall 1967

Section 53:

Article VII — an approximate quotient for the {(Z + 1/Z) dividend} Division.

Just below the "Break" the trial dividend was:—

```
15625 00' 000 00
160 4 6 0 3 306 25
```

Taking 9 as the next trial digit of the quotient:— Therefor the true value of the digit is 9 minus.

This agrees with Page 37. a; Section 53; Article VIII, comparison I; but both last digits are indefinite. For a very acurate value I extended the {(Z + 1/Z) dividend} quotient to 31 decimal places...

Article VIII — The {(1/Z) dividend} Division.

as the Divisor extends two places below the decimal point, we place the "caret" two places below the decimal point in the Dividend. The crossing of the Caret fixes the decimal point in the quotient.

```
                                          Point of Break
       1141051.24 ) 1.00‸0000000 (.00 00008763
                      91‸2840992  ) plus 84832
551428880              87159 0080
456420496              798735868
 950083840             728542120
 912840992             684630744
  372428480             439113760
  342315372             342315372
   301131080             967983880
   228210248             912840992
    72920832              55142888
```

Page 37.b

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Section 53:

article VII — The {(z + 1/z) dividend} Division.

Division of Remaining (Sm. Inc. V. l. S. C.) at the (Sm. Inc. V.) Rate by the Added Increase.

Point of Break

1141053.24 00008763

17,828,925,625) 20,343,753,350,140,900.
17828925625
2514827725 1
1782892562 5
731935162 64
713157025 00
187781376 40
178289256 25
9492120159 0
8914462812 5
5776573465 0
5348677687 5
427895777 50
356578512 50
71317265 000
71315702 500
→ 1.562500 1

quotient extends eight decimal places below point of Break.

last zero brought down

Beginning of Break.

156250000000
142631405000
136185950000
124802479375
113834706250
106973553750
68611525000
53486776875
15124748125

Continuation of Division

Continuation of Quotient → Extends 8 more places.
→ 84832639

151247481250
142631405000
86160762500
71315702500
148450600000
142631405000
58191950000
53486776875
47051731250
35657851250
113938800000
106973553750
69652462500
53486776875
161656856250
160460330625
1196525625

Remainder to Page 37.d

Page 37. da

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Section 53:

Article VIII — The {(1/2) dividend} Division, continued

```
1141051.24) 729208320      (plus 639067111482
            684630744             +21529
            ---------
            445775760
            342315372
            ---------
            1034603880        Two aughts.
            1026946116
            ----------
              76577648 0
              68463074 4
              ----------
              81145656 0
              79873586 8
              ----------
              127206920
              114105124
              ---------
               131017960
               114105124
               ---------
                169128360
                114105124
                ---------
                 550232360
                 456420496
                 ---------
                  938118640
                  912840992
                  ---------
                   252776480
                   228210248
                   ---------
                    245662320
                    228210248
                    ---------
                     174520720
                     114105124
                     ---------
                   604155960
                   570525620
                   ---------
                   336303400
                   228210248
                   ---------
                   1080931520
                   1026946116
                   ----------
                     53985404
```

Set over

Division continued

```
Divisor | Quotient
        |   +473119892
--------+-------------
539854040
456420496
---------
834335440
798735868
---------
 355995720
 342315372
 ---------
 1368034 80
 1141051 24
 ----------
  2269835 60
  1141051 24
  ----------
  1128784360
  1026946116
  ----------
   1018382440
    912840992
   ----------
   1055414480
   1026946116
   ----------
     2846838 40
```

Digits in the quotient of {(1/2) Division}
below point of "Break":

```
 8 or 9 } Page 37. fa
 5  + 4 }
--
13    12 }
     + 5 } Page 37. da
     + 9 }
otal  39
```

Page 37.d

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Union Fall 1967 Iowa

Section 53:

Article VII. – Continuing the {(Z+½) dividend} Division.
The Remainder brought forward = 1196525625.

additional Decimal places of quotient 067111482215294+

```
17,828,925,625 )  119652562500
                  106973553750
                  ------------
                  128790087500
                  124802479375
                  -------------
                    198760812500
                    178289256 25
                    ------------
                     204715562500
                     178289256 25
                     ------------
                      264263062500
                      178289256 25
                      ------------
                       859738062500
                       713157025000
                       ------------
                        1465810375000
                        1426314050000
                        -------------
                          394963250000
                          356578512500
                          ------------
                           383847375000
                           356578512500
                           ------------
                            272688625000
                            178289256 25
                            ------------
                             943993687 5
```

Sum of decimal places below point of "Break"

This Page	15
Page 37.b	8
" "	8
Total	31

Set away from margin.

```
9439936875 0
8914462812 5
------------
525474062500
356578512500
-------------
1688955500000
16046033 0625
-------------
  84352193750
  71315702500
  -----------
  13036491250
```

Page 37. e
Section 53:

Laurence M. Rash
Union, Iowa

The $\{(Z + \frac{1}{2})$ dividend$\}$
Division Quotient:

The $\{(\frac{1}{2})$ dividend$\}$
Division Quotient:

$$
\begin{array}{r}
\underbrace{1141053.24\ \ 0000876384832639067111482\,2+}_{\text{Point of Break}} \\
.00\ \ 00008763848326390671148\,22+ \\
\hline
1141053.24\ \ 00000000000000000000000000
\end{array}
$$

The two Parallel divisions on Pages 37. d and page 37. d. a. show that the decemials agree to 33 decimal places. Therefor, by subtraction, Z = 1141053. 24.

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Section 54:

February 3, I968

I have found three methods, all closely related, of finding the value of Z.

Method I -- The use of Intangibles alone. Pages 34 to 36.c .

Method II -- The use of Intangibles which disclose a point of Break. Pages 37 to 37.d .

Method III (a) -- The use of the point of Break to find the (Common Division Factor) of the Division.

Method III (b) -- When there is no point of Break, to find the approximate last decimal place of the value of Z , and the beginning of the value of I / Z; which sould lead to the value of the (Common Division Factor).

The value of Z and the value of (I /Z) may overlap and become a sum for a number of decimal places.

Methods I and II have already been explained.

Method III (A)_ Feb. I , I968 (II:30 P.M.)

I thought that I did some original work on this problem in August and September of I 967. But I now find the date May 1967. In the Spring and Summer I found employment helping erect the elevator at Beaman, Iowa. Since near Oct. I, 1967 I have worked at the ACCO seed corn company plant between Whitten and Conrad, Iowa . ~~seed corn company~~

Therefore when I began to reveiw my notes preparatory to proceeding with this book, I had completely lost my earlier sequence of thoughts. When I came upon this division I could not understand where I got the factor I5625 from. I now have a complete explanation.

```
15625)17,828,925,625.(1141051.24
      15 625
       2 2039
       1 5625
         64142
         62500
          16425
          15625
            80062
            78125
             19375
             15625
              37500
              31250
               62500
               62500
```

decimal point

Two digits brought down.

(15625) + (100) is the remainder
at the beginning of "Break."
Page 37. b.

Section 54 _ Continued:

I7,828, 925, 625 of course is the divisor in the {(Z + I / Z) DIvid-end} divisi-on. I5625 and Z are its factors.

Therefore I56 25 i-s a factor of both the Divisor and the Renainder, two places below the decimal p-oint, in the {(Z + I/ Z) DIVISION} division .. Therefore, from this point onward, the {(Z + I/Z) dividend} division is but the { (I/ Z) dividend} Division on a larger scale.

Feb. I, I96 8.

I call the numerical factor common to the DIVISor and the Dividend in the {(Z + I / Z) dividend} division, the Common Division Factor.
Page 37.b _ IN THE {(Z + I/ Z) dividend} division quotient a-t the "p oint of break" the value equals (Z + 2)in terms of Added Increase. The Remaind-er equals (I / Z) Added Increase = I162500. The two z-eros were brought down afte-r crossing the decimal point. The decimal point was crossed to complete the value of Z .

Feb. 3, I96-8.

Page 37. b .. The divisor is the numerical coefficient for Added I-ncrea-s-e .

T he Q-uotient, to the "Point of Break" = (Z + 2) . The-efore the Remaind-e-r I5 625 is the value which represents (I / Z). The two zeros were added after the division cros-sed the decimal point. This is the value of the " Com-mon Divi-sion Factor " in this case.
A-BSTRACTLY : The parts of the division ma y be reatated abstractly thus :
Divisor = (Common Division Factor) (Z).
From the dividend subtract the numerical value of the numerator of 2 a-fter placing it over the common denominator of the dividend and divisor.

Corrected QuOTIENT = (Z + I/ Z) .
A-s the corrected dividend = (DIvisor) (Quotient) ; then the Corrected di-vidend = (Common Division Factor) (Z) {COrrected Quotient} =
(Common Division Factor)(Z) (Z + I/Z) =
(COMMON Division Factor) (Z^2 + I). Repeating (Z^2 + I).

RELATED ABSTRACT VALUES:

The first Recognition Segregation Diagra-m, Page 33.a, shows the entire are-a i n terms of A-dded Increa-se, as equal to (Z + I/ Z). In which - Pa ge 3 I. a - Rem-ainder of (T.V.l.s.c.) at the (T.V. Rate) = Z .
Back to Page 33.a _
The two Increases = 2 and W = (I / Z).
Subtrac-t the two Increases from the entire area, leaving (Z + I/ Z) i n terms of Added Increase. This area also equals (Z^2 + I) in terms of W.

I-f the value of Z i s found, it gives Remaining (Sm. I nc. V L.S.C.) at the (T.V. Ra te) in term s of Added Increase. - Pa ge 3I.a . Change thi s value into term s of entire ± s. This a-rea is also measured in terms of G. Di-viding by the number of Remaining (Sm. Inc. V.) G , the number of ± s in one (T.V.)G is found. This opens the way to calcula-te the number of ± s i-n one (T.V.) ±. Thi-s gives the value of (T. V.) which i s tantamount to a solution of the problem .

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Section 54 — continued:

Feb. 27, I968.

The Two III. b Methods:

Note: While the III. a Method, (THE Common Division Factor Method), discloses some interesting relationships, the calculations of themselves, are not an improvement over Method II, the"Point of Break" method. The advantage is that it furnishes the theory for the two III.b Methods .

The two III.b Methods dig deeper into the problem, and I hope will be helpful when extracting the square roots of Surds..

METHOD III(b. a)

(Introduction) From Page 39: "I call the numerical factor common to the Divisor and the Dividend in the {(Z + I / Z) DIVIDEND} DIVISION? THE ~~[illegible]~~ Common Division Factor." On Page 38 the Common Division Factor is I5 6 25. In this case the Common Division Factor is an integer. If we were sure that it would always be an integer, we could find it as the nearest integer by dividing the Divisor by a close approximate of the value of Z. It occurs to me that in many cases the numerical value of both the Common Division Factor and Z will be a Mixed Decimal.

Both the Divisor and the Dividend of the {(Z + I/Z) dividend} division, will always be integers because they are placed over a common denominator. This will be true even with Surds.

Now the problem is to find pairs of decimals or Mixed Decimals whose products are integers only, thus moving the product to the left across the decimal point,

(I) — A multiplication between the terminal digits of two mixed decimals of one place, whose product is ten, or a multiple of ten, will move the terminal decimal digit one place to the left in the product. These pairs are:
.2 X .5 = .I ; .4X.5 = .2 ; .6 X .5 = .3 ; .8 X .5 = .4 .
There are no other pairs. ;

(2) — To find a first digit to the left of the decimal point, whose addition to one of the pairs above, will in the product give a sum of ten, and thus move the termination of the product an~~th~~other place to the left.

(a) In the above pairs, five can not be placed in either the multiplier or multiplicand because all products of five have either five or zero as their last digit; neither of which give a sum of ten in the addition. Thus: .I + .5 = .6 ;
.2 + .5 = .7 ; .3 + .5 = .8 ; .4 + .5 = .9 ..
I
In the pairs (I.5)X(.2) = .30 ; (I .5)X(. 4) = . 60 ;
(I . 5) X (. 6) = .90 ; and (I.5)X (8) = I.20 .
6

(b) — Replace (I . 5) by (2. 5) then (2. 5)X(. 4) = I.00;
and (2.5)X .8 = 2.00. The products are integers.

(C) — Decimal point moved three places to the left; (I.25) X .8 = I.000 .

(d) — It is possible that mixed terminals digits could move the terminal digits above zero farther to the left, but the addition of partial products becomes complicated.

Conclusion: AS the divisor and dividend will always be integers, therefore , with the four above exceptions, plus others, either the value of Z or the value of the Common Factor of the divisor and the dividend maybe an integer.

Section 54 - . continued:

Method I II. b.b.

Let us ignore the fact that we know about the "Break" in the (Z + I/Z) QUOTIENT ON Page 37.b and use it as an illustration. As Z is much greater than one, we are sure that (I/Z) is much smaller than one; therefore all the value in the integer part of the corrected quotient belongs in Z . As (I/ Z) is one divided by this relatively large integer, it is a very small number. Because of the small decimal to the right of the decimal point, we know that the value of Z is but little larger than II4 I05I ; and smaller by (I/Z) than (Z + I/Z). These two values form a pair which limits the area in which the value of Z is possible.

First consecutive trial division - To find in which decimal p-lace the value of (I/ Z) begins:

The trial division is calculated on Page 42.

Two facts have been established:

(I) The value of (I/Z) begins in the seventh place below the decimal point.

(2) Because of the agreement of the two quotients, we know that there is no overlap of value to become a sum, between (Z) and (I / Z) down to and including the eleventh decimal place. Therefore the true value of Z is found including the eleventh decimal place.

Insert note → In order to have a method which will solve surds, let us change the corrected (Z + I/Z) quotient to read II4I05I.24????. (LET ? be the symbol for a digit of unknown value.) Then we are sure the value of Z equals II4I05I.24???? to six decimal places.

Second consecutive trial division:

Divide one by this more accurate value for Z. Extend the division to about fifteen places below the decimal point. Because the value of Z is only to six decimal places, it is possible that there are additional decimal places in the true value. Then the true value of Z is slightly larger than this approximation. For this trial division, this is the bottom boundary of the area of the possible valueof Z . Because the value of (I/Z) varys inversely, the value found for (I/ Z) by using this more accurate value of Z for the divisor, is slightly larger than the true value. This is the top boundary of the area of the possibility of the true value of (I/Z).

F-or the other part of the pairs of values:

Because of possible a-dditional decimal places the value of Z may be larger than the appromination. Add one to the last digit of this more accurate value for Z. We know that now this value is larger than the true value of Z, becaues one is larger than any decimal faction which follows it. Inversely the value found for (I/Z) is slightly smaller than the true value. This is the bottom boundary of the area of p-ossibility of the true value of (I/Z).

These two boundaries [illegible] contain the possibilities of the true value of (I /Z) so close together that their values will be id-entfal for a number of decimal ~~pl~~ pla-ces. The decima-l places which are in agreement are those of the true value of (I/ Z). Subtract this value from the value of (Z + I/Z) to find a more accurate value for Z.

Third consec-utive trial division:

Use this last value for Z to find a more accurate value for (I/Z).

Repeat these consecutive trial divisions as often as desired..

An interesting variation is : Divide the divisor of the {(Z + I/Z) dividend } division by the value found for Z. This gives the (Common Division Factor). Divide the corrected Dividend by this (Common Division Factor). That gives this same area as (Z^2 + I) in terms of W .

Section 54 – continued ,
Method III . b.b. – note t-o be inserted on Page 4I :

I-n case there is a disagreement between the two quotients either; (I) the division has been overextend-ed because the divisor i-s only a-n app-roxima-tion ~~be~~ of Z ; or (2) there is an overlap between the values of Z and (I / Z).

SECOND NOTE: I BELIEVE THIS I-S THE END OF MY THEORY.

Following is the First consecutive trial Division – To find in which decimal place the value of (1/2) begins. Explanatory to Page 41.

value of (1/2) begins

end of agreement

```
                 .000000 87638 5
1141051 ) 1.0000000
           9128408
           8715920
           7987357
            7285630
            6846306
             4393240
             3423153
              9700870
              9128408
               5724620
               5705255
                 19365
```

"End of agreement" with the quotient of {(Z+1/2) dividend} division – Page 37. b. The reason is that the present divisor is only an approximation and the division is over extended.

Part III –

Results of the Theory.

Chapter VIII

Section 55

Page 42½

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Reference List of meaning of Terms and operations Used in Chapter VIII.

Page 43 — $(a^2 - n^2)$ — References (conception, page 3.b), (bottom page 5) (and page 5.b), (Section 14, page), (page 17, Section 37, Article 2).

"Setting Up" Integral Series and Diagram.
Page 17, Section 37; Article 2 and also Procedure.
Page 6.b, and Page 6, Pages 5 and 5.b.
Page 18 and ½ Section 37 and ¾.

Subtraction of number of divisions in Integral Diagram from Complete Diagram — Page 16, Section 37 — Article I. also Page 18.a.

Page 43.a — Two Calculations of (L.S.C.) — Page 18.e Section 37 and ½.
Analysis of L.S.C. — Pages 20, 20.a — Section 39.

Page 43.b — Introduction of (Sm. Inc. L.S.C.).
Refinement of (Ls.S.C.) — Pages 21, 21.a, and 21.b all in Section 40.
Analysis of G-Space — Pages 22, 22.a, 23.a, and 24.

Note: There are as many ±s in L, as there are Gs. in (l.S.C.) — bottom of Page 24.

(Sm. Inc. L.S.C.) — Page 27, Section 46.

Page 42½.a

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Reference List—

Calaulation of (Sm. Inc. V.(l.Sc.)) — Page 30.c, Section 48, Article III, Subarticle 3.

Page 43.c — Remaining (Sm. Inc. V.) l.s.c. at the (Sm. Inc. V) Rate — Page 32, Section 50.

Page 43.d — The $(Z + \frac{1}{2})$ dividend Division.
Page 32, Section 51 — The Z Theory.
Page 33, Section 52 — Intangibles,
Segregating Diagrams — Page 33.a, Section 52
Page 34, Section 52, Set I — The first Intangible coefficient, $(Z + \frac{1}{2})$.
Page 41, Section 54, Method III. b. b.

Pages 43.d to Page 43.g, (Section 55: Article VI to Article IX) — The two parallel divisions, finding the value of Z by the subtraction of quotients.

References: all of Section 53, beginning at the top of Page 36:
Basic Calculations Page 36 to Page 36.c.
The two parallel divisions Page 37 to Page 37.d.a.

Note: as I had calculated the value of Z, I did not continue further. Page 42 is the last page which I refer to. This is the end of my theory. The rest of the calculations are only the solutions of quantities made possible by the Theory

(next Page)

Page 42 $\frac{1}{2}$. b

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List of Contents —

Page 43. h and Page 43. I —
Calculation of the number of Added Increases in one G (at the T.V. Rate).

Page 43. j — To change the number of Added Increases in one G, into complete $£^{s}$.

Page 43. K — Calculating T. V.

Page 43. l — Comparision between the value of (T. V.) and (Sm. Inc. v.).

Page 43. m — a recheck on the number of d differences in the Division furthest right.

Note: This is a recheck on the comparision

Pages 43. n and 43. o — Article XV —
To find the Ratio of d: (l. s.).

Page 43. p — Coefficients for N, and for R.

Pages 43. q, 43. q. a, and 43. q. b — The "Grand Division." The quotient is the $\sqrt[2]{8}$ or N.

Laurence M. Rash
Union, Iowa
June 25, 1968

Page 43. a

Section 55:

Article II – Introduction of G values.

If the (l.s.c.) were just below n^2, it would equal $(l.s.)^2 + 32$ ch.

There are $(14 + \frac{50}{33})$ ch beneath the Integral diagram, which equals $(15 + \frac{17}{33})$ ch.

$$(32 \text{ ch}) - (15 + \frac{17}{33}) \text{ ch} = (16 + \frac{16}{33}) \text{ ch}$$

$$(L.s.c.) = (l.s.)^2 + (16 + \frac{16}{33}) \text{ ch}.$$

$$= (l.s.)^2 + (\frac{544}{33}) \text{ ch}.$$

16
33
48
48
528
+ 16
544

Place all the ch in (l.s.c.) above the two rows of ch.
$(a^2 - n^2)$ is 33 d high. The column of $(\frac{544}{33})$ ch will have a width of $(\frac{544}{1089})$ l.s.
(note: a little less then one half – Page 20)

33
33
99
99
1089

Dividing by 33:

(Page 20) S will equal $\frac{1088}{1089}(l.s.)^2$, and $G = \frac{1}{1089}(l.s.)^2$.

$2 \times \frac{8}{33}$ ch $= \frac{16}{33}$ ch $= X$ ch. (Page 20.b)

$(\frac{544}{33}$ ch$) \div (\frac{16}{33})$ ch $= 34 =$ number of X ch above the two rows of ch.

There are 34(S) + column of K s plus L in (l.s.c.) above 2 rows of ch.

16)544(34
48
64

S = 1088 G.

34 S + one S = 35 S.

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Page 43. b
Section 55:

1088 G
35
5440
3264
38080 G
lora G 1
38081 G

June 26, 1968.

The (l.s.c.) = 38081G + column of K s.
There are 34 χch above the two rows of ch = X.
" " 35 χch in all = y.

Article III - Introduction of (Sm. Inc. L.S.C.).
(value of T.V. is tantamount to a solution of L.S.C.)
article III, Page 21.)
(Sm. Inc. L.S.C. - Page 27, Section 46
also Diagram 2, Page 23.a)

The Sm. Inc. L.S.C. = 38081 G + $\frac{34}{35}$ G.
($\frac{34}{35}$ Ⱡ has been added) Page 30.c.
Equals $\frac{1,332,869}{35}$ G.

38081
35
190405
114243
34
1,332,869

Article IV - Calculation of one G in values of Ⱡ. (Page 36)
There are $\frac{1,332,869}{35}$ Ⱡ in each L.
There are $\frac{1,332,834}{35}$ Ⱡ in each Ⱡ.

we divide by 34, and multiply 35 to include the K beneath each L.

$\frac{\cancel{35}}{34} \times \frac{1,332,834}{\cancel{35}}$ Ⱡ = $\frac{1,332,834}{34}$ Ⱡ. (Page 36.a

Page 43. C
Section 55:

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add the lone £ = $\frac{1,332,868}{34}$ £ = one (Sm. Inc. V.) G at the (Sm. Inc. V.) Rate.

Article V — Calculating the number of £ s in Remaining (Sm. Inc. V. l. s.c.) at the (Sm. Inc. V.) Rate.

There are $\frac{1,332,869}{35}$ G in (Sm. Inc. l.s.c.). minus $\frac{1}{y}$ G = $\frac{1332868}{35}$ = {Remaining (Sm. Inc. V. l.s.c.).

(note: The numerators are the same.)

$\frac{1,332,868}{35}$ X $\frac{1,332,868}{34}$ = number of £ s in Remaining (Sm. Inc. V. l.s.c.) at the (Sm. Inc. V.) Rate.

$$\begin{array}{r} 34 \\ 34 \\ \hline 136 \\ 102 \\ \hline 1156 \end{array} \qquad \begin{array}{r} 35 \\ 34 \\ \hline 140 \\ 105 \\ \hline 1190 \end{array} \qquad \begin{array}{r} 1,332,868 \\ 1,332,868 \\ \hline 10662944 \\ 7997208 \\ 10662944 \\ 2665736 \\ 3998604 \\ 3998604 \\ 1332868 \\ \hline 1,776,537,105,424 \\ 0\,1\,2\,3\,4\;3\;3\;3\;1\,1\,1 \end{array}$$

(added Increase)

(common denominator)

(numerator) →

(carry over) →

$\frac{1,776,537,105,424}{1190}$ £ = Remaining (Sm. Inc. V. l.s.c.) at the (Sm. Inc. V.) Rate.

$\frac{34}{35}$ £ = $\frac{(34)^2}{(35)(34)}$ £ = $\frac{1156}{1190}$ £ = added Increase.

Page 43.d

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Section 55:

Article VI – The (Z + ½) dividend Division.

(Refference page 37. f.)

Note: as both numerators are over the same common denominator, 1190, only the numerators appear in the division.

```
1156) 1,776,537,105,424 (1536796804
      1156
      6205              11190
      5780              10404
       4253              7865
       3468              6936
        7857              9294
        6936              9248
         9211              4624
         8092              4624
         1119
```

In terms of Added Increase,

$1536796804 = (Z + 2 + \frac{1}{2})$.

$\underline{\qquad\qquad - 2}$ = Subtracting the two fractional Z s.

$1536796802 = (Z + \frac{1}{2})$.

Article VII – a discussion of the method of separating Z and ½.

(Sunday, 1:45 A.M., Dec. 8, 1968.)

This proof is the establishment of the approximate relative values of Z and ½ in the sum (Z + ½); so that I may proceed with the ½ dividend Division.

Section 55

Article VII continued:

I - The normal case — when the value of Z is greater than one.

Given the value of Z as equal to one.
Then $(Z + \frac{1}{Z}) = (1 + \frac{1}{1}) = 2$.
Therefore when $(Z + \frac{1}{Z}) = 1,536,796,802$; the value of $(\frac{1}{Z})$ would approximate the very small value $\frac{1}{1,536,796,802}$ as a limit.

as the value of Z must be smaller than $(Z + \frac{1}{Z})$ I have taken Z as equal to 1,536,796,801. also this value will produce a slightly too large value for $(\frac{1}{Z})$. This makes sure that all value in the remainder, to the left of the "caret," belongs in Z.

(2:30 A.M., Dec. 17, 1968)

We know this case is a normal case because the number of G spaces in the (l.s.c.) is a large number. The number of fractional £ s, which is an approximation to Z, is greater than the number of £ s in all G s.

II - There is an abnormal series of values also.

Suppose $Z = \frac{1}{3}$

Then $(Z + \frac{1}{Z}) = (\frac{1}{3} + \frac{1}{\frac{1}{3}}) = (\frac{1}{3} + 3) = 3\frac{1}{3}$

This case is entirely theoretical. It would not be practical to use a diagram which would produce such a value for $(Z + \frac{1}{Z})$.

Page 43. f

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Section 55:
Article VIII — The 1/2 dividend Division.

we know Z is less than 1,536,796,802.
we use:

```
1,536,796,801) 1.0000+         (.000 000 000 continued
               9220780806        below
               7792191940       +65070411348+
               7683984005            ↑
               10820793500      18th decimal
               10757577607        place.
                632158930 0
                614718720 4
                174402096 0
                153679680 1
                207222159 0
                153679680 1
                535444789 0
                461039040 3
                744057487 0
                614718720 4
               12933876660
               12294374408
```

Note: we are working only with the first intangible values. This is the (first consecutive trial division) of method III. b. b. Section 54, Page 41

The subtraction of 1/2 from (Z+1/2).

```
1,536,796,802.
minus            .000 000 000 650 704 113 48+
1,536,796,801.999999999349295886 52-
                        ↑
```

we are sure all value to the left belongs in the value of Z, because the value of 1/2 begins in the tenth place below the decimal point.

Note: Even 1,536,796,802 used as a divisor would not effect the quotient until the 18th decimal place. See page 43.g. Therefore a second division is not needed unless the calculation is to be carried to

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Dec. 23, 1968

Page 43. g.
Section 55.

Article IX — The 1/Z dividend Division, using the more accurate value for Z.

```
1,536,796,801.999,999,999) 1.000,000,000,000 000 0000  | .000 000 000
                             9220780811999 999 994      | +650704
                           ---------------------------- | +1130
                              779219188000000 00060         ↑
                              768398400999999 99995        18th
                              ---------------------        decimal
                              108207870000 0 0006.500      place
                              107575776139 9 9999.993
                              -----------------------
                               063209386 00 000650700
                               61471872079999 99999 6
                               ----------------------
                                1737513920006507040
                                1536796801999999999 9
                                ----------------------
                                 200717118006507 0410
                                 1536796.801 9999999 99
                                 ----------------------
                                  47037437806507041 10
                                  4670390405999999 9.997
                                  ----------------------
                                   933533746507041130
```

$(Z + 1/Z) = 1536796802.$
$\qquad .000000000650704113+$

$(1/Z) =$

$(Z) = 1536798 01 .999999999349295886+$

note: The last digit of the value of (Z + 1/Z) is 2 followed by a row of zeros below the decimal point. The subtraction of the least value, will change the digits to 1+ a row of nines as a decimal fraction. Therefore the end of the value of Z is given as 6+.

The two quotients (this page, and page 43-f.) are two approximate values of (1/Z). They agree to 18 decimal places. That is close enough for the present purpose. — Dec. 24, 1968.

The next digit in the second quotient (which is the more exact calculation) is zero. Therefore we are sure of the value of Z to 18 decimal places.

note: we are working only with the first intangible values. This is the (Second consecutive trial division) of method III. f. f. Page 41.

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Dec. 27, 1968

Page 43. h

Section 55

article X – To calculate the number of Added Increases in one G (at the T.V. Rate).

In terms of Added Increase, Z is taken as 1,536,796,801.999,999,999,349,295,886+.
Then the value of (Remaining Sm. Inc. V) at the T.V. Rate = 1,536,796,802.999,999,999,349,295,886+.
There are in the (Remaining Sm. Inc. l.s.c.) $\frac{1,332,869}{35}$ G $- \frac{1}{y}$ G $= \frac{1,332,868}{35}$ G. Page 43.c.

Divide 1,536,796,802.999,999,999,349,295,886+

by $\frac{1,332,868}{35}$.

1,536,796,802.999,999,999,349,295,886+
35
7683984014.999999996746479430
4610390408999999998047887658
53,787,888,104.999,999,977,225,356,010+

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Page 43. I
Section 55.
Article X - Continued - Division continued.

```
1,332,868) 53,787,888,104·999,999,977,225,356,010 +
           5331472
            4731681                    11136572
            3998604                    10662944
             7330770                    4736282
             6664340                    3998604
              6664304                    7376785
              5331472                    6664340
              13328329                    7124453
              11995812                    6664340
               13325179                    4601135
               11995812                    3998604
                13293679                    6025316
                11995812                    5331472
                 12978679                    6938440
                 11995812                    6664340
                  9828679                     2741001
                  9330076                     2665736
                   4986039                      752650
                   3998604
                    9874359
                    9330076
                     5442837
                     5331472
                      111365
```

quotient
40354·9999
+73,740,835,
+534,520+

note: The entire quotient of this division equals the number of Added Increases in one G (at the T. V. Rate).

Page 43. j

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Dec. 28, 1968

Section 55:
article XI – To change the number of Added Increases in one G into complete ± s.
The Added Increase = $\frac{34}{35}$ ±.

Therefore 35 Added Increases would equal 34 ±.
For $35\left(\frac{34}{35}\right) ± = 34(±)$.

There are $\frac{34}{35}$ as many complete ± s.

```
         40354.999,973,740,835,534,520+
                                     34
       ----------------------------------
         161419999894963342138080
        1210649999212225066603560
       ----------------------------------
35) 1372069,999,107,188,408,173,680+
    105
     322
     315
     070
      70
      0069
        35
        349
        315
         349
         315
          349
          315
           341
           315
            260
            245
             15

157
140
171
140
 318
 315
  38
  35
  340
  315
   258
   245
    131
    105
     26

267
245
 223
 210
  136
  105
   318
   315
    30
```

quotient below:
39201.999,974,
491,097,376,390+

Page 43. K.

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Union, Iowa

Section 55:

Article XII – To calculate the number of ₤ s in L, which equals T.V. .

Subtract the lone ₤.

```
     39201.999,974,491,097,376,390+
   -     1.
   ---------------------------------
35)39200.999,974,491,097,376,390+ = ₤ s in T.V. (₤ + K)
   35
    42
    35
     70
     70
     00099
        70
       299
       280
        199
        175
         247
         245
           2

  244        197        89
  210        175        70
   349        223       190
   315        210       175
    341        137
    315        105
     260        326
     245        315
      159        113
      140        105
       19          8

quotient below
1120.028,570,
+699,745,639,
+325+
---------------------------------
  1120.028,570,699,745,639,325+ = { ₤ s in one K at the (T.V.) Rate }
                             34
  4480114282798982557300
 3360085712099236917975
---------------------------------
 38080971,403,791,351,737,050+ = { ₤ s in one ₤ }
     1.                 adding lone ₤
---------------------------------
 38081.971,403,791,351,737,050+   ₤.
```

This is the number of ₤ s in (T.V.) L.

This also equals the number of G s in (T.V.l.s.c.).

Note: See Page 22, Section 41, Article 1.

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Page 43. l.
Section 55:

Article XIII— Comparision of the value of (T.V.) with the value of its close approximate, (Sm. Inc. V.).

These values are so close together that their integral parts will be identical. What we are comparing are the decimal parts of these values.

Reduce the common fraction 34/35 to a decimal fraction:

```
35) 34.0                    100        .9714285 + the
    31 5                     70        repetend 714285+
     250                     300       repeated without end.
     245                     280
       50                     200
       35                     175
       150                     25
       140
        10
```

Thus the (Sm. Inc. V.) G value varys from the (T.V.) G value, only after the fourth decimal place. [The (T.V.) G value, page 43. K]. I didn't realize how minute a quantity Added Increase was.

I believe that my Extensive calculations are correct.

The (Sm. Inc. V.) is a very close approximate of (T.V.). Except for very exact calculations, all of this striving after the exact value of (T.V.) is a waste of effort.

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Section 55:

Article XIV — a recheck on the number of full d differences in the Division furthest right.

The 8th difference $= (2v-1)d^2 = 15d^2$.

The last division in the Integral diagram $= \frac{25}{33}$ division

$$\begin{array}{r} 33 \\ \underline{7} \\ 21 \\ \underline{21} \\ 231 \\ \underline{25} \\ 256 \end{array} \qquad \frac{256}{33} d^2 = \text{row of } d^2 \text{ in the Integral diagram.}$$

$$\begin{array}{r} 15 \\ \underline{33} \\ 45 \\ \underline{45} \\ 495 \end{array} \qquad \frac{495}{33} d^2 = 8\text{th difference.}$$

as all calculations are over the same denominator, omit it from the calculations.

For the top $\frac{25}{33}$ division space;

$$\begin{array}{r} 15\ d^2 \\ \underline{25} \\ 75 \\ \underline{30} \\ 375\ d^2 \end{array} \qquad \begin{array}{r} 495 d^2 \\ \underline{-375 d^2} \\ \frac{120}{33} d^2 \end{array} \text{ needed}$$

This must be subtracted from the row of d^2's.

$$\begin{array}{r} 256 \\ \underline{-120} \\ 136 \end{array} \text{ Remaining in row of } d^2\text{s.}$$

Thus the 1st full d difference is completed.

9th d difference $= (2v-1)d^2 = 17d^2$.

The Big Space is calculated correctly. There are 2 full d differences in the $\frac{25}{33}$ division + the row of d^2 of the Integral diagram.

$$\begin{array}{r} 17 \\ \underline{33} \\ 51 \\ \underline{51} \\ 561 \\ \underline{-425} \\ 136 \end{array} \qquad \begin{array}{r} 17 \\ \underline{25} \\ 85 \\ \underline{34} \\ 425 \end{array}$$

from fraction of space

561 — entire difference

136 balance of row of d^2s!

Then $(a^2-n^2) =$ the 17th difference $= 33d^2$.

The diagram has a height of 33 d above the 2 rows of ch.

Compare with page 43.

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Section 55:

: l.s.
Article XV - To find the Ratio of d : l.s.

(The calculations and digrams for this rearrangement are on pages 20. to 20.c; all under Section 39.)

(J.V. l.s.c.) = 38081.971, 403, 791, 351, 737, 05$\overset{G}{0}$.
Subtract the lone G -1.

Equals 38080.971, 403, 791, 351, 737, 050 + G

This is the number of G s in a column whose height is (33 d + 2 l.s.) - Article XIV, this Section; and whose uniform width is $\left(\frac{544}{1089}\right)$ l.s. - Page 43.a, Section 55, Article II.

As this column is of uniform width the height of any segment of the column is measured by the number of G s it contains. Thus S is 2 l.s. in height, and contains 1088 G's. Therefore (l.s.) is measured by 544 G's.

S = 1088 G
38080.971, 403, 791, 351, 737, 050 G
1088. G - Subtract S.
36992.971, 403, 791, 351, 737, 050 G

The column whose height is 33 d, is measured by 36992.971, 403, 791, 351, 737, 050 G.

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Page 43. o.
(which is page 43. n – continued.)

Article XV – continued.

Divide the above G value of the above Column by 33, the d coefficient of the height.

33) 36992.971,403,791,351,737,050 + G

33
39
33
69
66
329
297
327
297
301
297
44
33
11

110
99
113
99
147
132
159
132
271
264
73
66
7

75
66
91
66
257
231
263
231
327
297
300
297
3

35
33
20

quotient below
1120.999,133
+448,222,779,
+910 +

Therefore one d is measured by
1120.999,133,448,222,779,910 + G;
Therefore d is to (l.s.) as
1120.999,133,448,222,779,91+ is to 544.

Article XVI – To find the value of N.

Information taken from this Section, article I and article XIV:

$N = 16d + (l.s.)$;
$A = 3 = 17d + (l.s.)$.

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Section 55:
Article XVI – continued.

(A) Calculating the coefficients for N, and for A, as measured by values of G.

1120.999,133,448,222,779,91+
16

67259948006893366 7946
1120999133448222779991

17935,98 6,135,171,56 4,478,56
544. ← add ½ (S)

18479.986,135,171,56 4,478,56 ← for N.
1120.999,133,448,222,779,91

19600.985,268,619,787,258,47 ← for A.

(B) The proportion is

Coefficient of A : Coefficient of N = 3 : {numerical value of N}

The product of the means = product of the extremes.

18479.986,135,171,564,478,56
3

55439.958,405,514,693,435,68 = the product of the means.

(C) Following on page 43.q, is the division of this product by the coefficient of A to find the numerical value of N.

Page 43.g.
Section 55:
Article XVI – continued.
Sub~~tricle~~-article (c) – "The Grand Division"

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Divisor — 19600.985,268,619,787,258,47

Dividend 55439.958,405,514,693,435,68

39201 970537239 574 51694

16237 98786827511891874 0

15680 78821489582980677 6

55719965337928911 1964 0

3920197053 7239574 51694

16517994800689336 6794 6 0

15680788214895829806 776

8372065857935068726 84 0

784039410744791490 33 88

531671750487153823452 0

392019705372395745 1694

13965204511 475 807828260

1372068968803385 1080929

2445148234419567473310

1960098526861978725847

48504970755758874 74630

392019705372395745 16 84

930300021851930022936

quotient below:
2.828,427,12

Page 43.q.a.
Section 55:
Article XVI –
Subarticle (C) – Continued

Divisor — 19600. 985,268,619,787,258,47

9303000218519300229360
7840394107447914903388
14626061110713853259720
13720689688033851080929
9053714226800621787910
7840394107447914903388
12133201193521068845220
11760591161171872355082
3726100323491964901380
1960098526861978725847
17660017966299861755330
17640886741757808532623
19131224542053222707000
17640886741757808532623
14903378002954141743770
13720689688033851080929
11826883149202906628410
11760591161171872355082
6629198803103427332800
5880295580585936177541
748903222517491155259

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quotient below
+4,746,190,09
7,603

Page 43.g.b.
Section 55:
Article XVI —
Subarticle (c) — Cont.

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```
Divisor — 19600.985,268,619,787,258,47
      748,903,222,517,491,155,2590
      588'029'558'058'5936177541
       16087366445889753750490
       15680788214895829806776
         4065782309939239437140
         3920197053723957451694
          14558525621528198544600
          13720689688033851080929
             8378359334943474636710
             7840394107447914903388
```

Quotient below

+3,82074

note: This is the end of correct calculation. This calculation enables me to calculate $\sqrt{2}$ correctly to 22 decimal places.

My reference authority is "Ripley's new Believe it or not!" page 207. This is accuracy to 5 more decimal places than in the divisor.

The reason for this limitation of accuracy is found on page 43.g. Here the value of Z is calculated to only 18 decimal places. The correct extention of this value would increase the accuracy of the two square roots ($\sqrt{2}$ and $\sqrt{8}$); but would involve dealing with decimals of a large number of digits.

IX

Chapter VIII – Extractions of $\sqrt[3]{2}$.

Section 56:

Relationships Between Different Sets of Diagrams for the same n^2. I am only calculating Sets of Complete and Integral Diagrams.

Article I – If we use integers, which are perfect squares, as the value of A^2:

Subarticle (a):

Let $A^2 = 9$, and $n^2 = 2$.

$(a^2 - n^2) = 7$, $n^2 \div (a^2 - n^2) = \frac{2}{7}$.

In the division of the complete diagram to the right of n^2: $2 \times \frac{2}{7} d^2 = \frac{4}{7} d^2$. Plus $\frac{2}{7} d^2$ from the row of d^2s $= \frac{6}{7} d^2$. This sum does not even equal the first d difference. Therefor there is only a (l.s.) difference below n^2. This diagram does not qualify as either a customary complete or Integral Diagram. This value for A^2 is larger than the limit of possible values.

Subarticle (b): There is only one remaining set of values. Let $A^2 = 4$; and $n^2 = 2$. Then $(a^2 - n^2) = 2$, and $n^2 \div (a^2 - n^2) = 1$. There is one division below n^2 and "by sight," two full d differences. Therefor $n^2 = 4d^2$; and $(a^2 - n^2) \equiv 5d^2$. $4 \div 5 = \frac{4}{5}$. There is $\frac{4}{5}$ of a division in the Integral Diagram below n^2.

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Calculation of the $4/5$ of a division in the Integral Diagram below n^2.
1st difference $= d^2$. $4/5$ of $d^2 = 4/5\ d^2$.
Plus $1/4\ d^2$ from the row of d^2s in the Integral Diagram — completes 1st difference
2nd difference $= 3d^2$.
$4/5$ of $3d^2 = 12/5\ d^2 = 2\,2/5\ d^2$.
add the remainder of row of d^2s in in the $4/5$ division, which is $3/5\ d^2$.
$2\,2/5\ d^2 + 3/5\ d^2 = 3d^2 =$ 2nd d difference.

Note: Explanation of my "By sight" calculations: In the present Example, there is one full division below n^2. The first d difference equals $1d^2$. There is $1d^2$ in the row of d^2s — not enough to equal the second d difference, which would be $3d^2$s. Therefor, here, the row of d^2s equals an (l.s.) difference.

As the row of D^2s only increases $1d^2$ with each succeeding Division, whereas the succeeding d differences increase $2d^2$; then in cases where the complete diagram is expressed entirely in full d divisions, the row of d^2 can only equal the "Little Space"

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Section 56: Article II – If we use integers, which are not perfect squares, as the value of R^2.

Subarticle (a):

Let $a^2 = 8$ and $n^2 = 2$, and $(a^2 - n^2) = 6$
$2 \div 6 = \frac{1}{3}$. This is an Integral Diagram.
Proof: $3 \times \frac{1}{3} d^2 = 1 d^2 = n^2$
$(a^2 - n^2) = 3 d^2$. Then $1 d^2 = 2$.

note: multiplying by 2, we have an R^2, which is a perfect square. $a^2 = 16$ and $n^2 = 4$. $(a^2 - n^2) = 12$. $4 \div 12 = \frac{1}{3}$. Therefor $d^2 = 4$. an Integral Diagram.

Subarticle (b):

$a^2 = 7$, $n^2 = 2$. $(a^2 - n^2) = 5$. $2 \div 5 = \frac{2}{5}$.
$2 \times \frac{2}{5} d^2 = \frac{4}{5} d^2$. add $\frac{1}{5} d^2$ to complete the first d difference. $n^2 = 1d + (l.s.)$.
multiplying by 7. $a^2 = 49$, $n^2 = 14$.
$(a^2 - n^2) = 35$. $14 \div 35 = \frac{14}{35} = \frac{2}{5}$
The same Integer Diagram.

Subarticle (c):

Let $a^2 = 6$, $n^2 = 2$, and $(a^2 - n^2) = 4$.
$2 \div 4 = \frac{1}{2}$. The second d difference below n^2 can not be completed. Integral Diagram $= \frac{1}{3}$.
multiply by 6. $(a^2 - n^2) = (36 - 12) = 24$.
$12 \div 24 = \frac{1}{2}$ = coefficient of complete Diagram.
$d^2 \div 3d^2 = \frac{1}{3}$ = coefficient of integral Diagram.

Subarticle (d):
Let $a^2 = 5$, and $n^2 = 2$. $(a^2 - n^2) = 3$.
$2 \div 3 = 2/3$ = complete diagram.
First difference $= 2/3\, d^2 + 1/3\, d^2 = d^2$.
For 2nd difference $2/3 \times 3d^2 = 2d^2$. There is only $1/3\, d^2$ left in the row of d^2s. Second difference is not completed.
multiply by 5:
$a^2 = 25$, $n^2 = 10$. $(a^2 - n^2) = 15$.
$10 \div 15 = 2/3$.

note: For each of these sets, there is a multiple set which has an R^2 which is a perfect square.

article III – If we use mixed numbers as the value of R^2.
Let $2\frac{1}{4} = a^2$, and $n^2 = 2$.
Then $9/4 = a^2$, and $n^2 = 8/4$.
If we extract the square root of 8, we can find the square root of 2, by dividing by the square root of 4.
$\sqrt[2]{8}$ below:

End of accuracy

$2\,\overline{)\,2.828,427,124,746,190,097,603,3\ 820}$

$1.414,213,562,373,095,048,801,6+ = \sqrt[2]{2}$,

This is the calculation of Chapter VI – "The Trial by Fire."

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Dec. 3I, I968

Chapter X
RESUM-E' and P-ROJECTIONS

Section 57:
Barriers

A — Inter-Dimensional (Between differente dimensions)

I think the great barrier between adjoining continua is that the unit of measure of the lower continuum can not increase its self laterally and become a unit of measure of the next higher continuum. Thus an infinite number of one inch length units lain side by side will never become one square inch unit. They do not have any width, and theoretically, do not cover the surface at all. A lower continuum does not approach so near the next higher nontinuum that it is transformed into it. That is impossible. But at any time it can be superinposed on the next higher continuum. Not only that, but a continuum can not exist without the lower continua , which make it possible to exist. Suppose we have a surface four feet long. It can not exist as a surface unless it has breath.

B — Intra-Dimensional (With in - Dimensional)

When dealing with Surds:

In the first dimensional, the relative values of (d) and (l.s.) are unknown. Here is a barrier. This barrier reappears in the second dimensional; the ratios of values between $(d)^2$, $(l.s .)^2$, and ch; also the height and lngth of the complete diagram; also the ratios between L and K; xch, K, and S; are unknown.

There are two classes of values which enter into these Intra- Dimensional relationships.

(I)— The abstract, qualitive, or theoretical. The primary ratio (D) : (L.S.) can only be known after a sOlution. (With Surds, never can this barrier be crossed short of a complete solution.) The height and length of the complete diagram are another variation of this ratio.

This ratio govens the relative values of $(d)^2$, $(l.s.)^2$ and ch in the first complete diagram, but there are no numerical relationships.

When the (l.s.c.) is rearranged and the value G is introduced, then some numerical relations are known, but nothing that discolses the primary ratio (d): (l.s.).

A secondary complete diagram can even be constructed, many of the above relationships incorporated, but the numerical value of the height and length can never be found exactly, short of a complete solution.

Abstract facts or relationships that exist in one dimensional, govern the abstract value of facts or relationships in another dimensional continuum.

The rule is: All diagrams of any given dimensional continuum are the result of all related values of all lower dimensional continua. Thus a cube equals a face (surface or right cross section) times a edge. A surface is a cross section of a cube. Any dimensional continuum has access to all corresponding values of a lower continuum because the lower dimensional continuum is only a face or cross section.

By inference, intelligent beings of one dimensional world are in possession of all the knowledge of beings of a lower dimensional world.

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Section 58;

The Spiritual and the Natural:

Where does the reader think the creative ideas for this book came from? I think they came from a higher dimensional continuum, from beings of greater intelligence than human -- the world of spirits. For instance, the "funny little table". Then also, the other essential, creative, ideas which came slowly, one at a time.

The Bible -- St. Matthew I8:

Verse I8 : Verily I say unto you, Whatsoever ye shall bind on earth shall be bound in heaven: and whatsoever ye shall loose on earth shall be loosed in heaven.
Verse I9 ; Again I say unto you, That if two of you shall agree on earth as touching anything that they shall ask, it shall be done for them of my Father which is in heaven.

Verse 20 : For where two or three are gathered together in my name, there am I in the midst of them.

(II) -- The quantitive, numerical, of known.
Under this condition all intra- dimensional barriers disappear -- as when working with a perfect square, whose N is known.

C -- Conversion of abstract values (I) into numerical values (I I).
(Refer to the Segregating Diagram, Section 49, Page 3I.a)

The secret of the success of the Intangibles is that there is only one intra-dimensional ratio -- W is I/Z of Added Increase, and Added Increase is I /Z of Remaining (T.V.l.s.c.). From this, Intangibles open the way to find (T.V.). This opens the way to find all values of the diagram of a surd. The introduction of Intangibles reduces all ratios in the solution to (Z : I). This key to measurement reduces a surd to approximately the status of a perfect square. Whenever the numerical value of N is known, all values can be found.

Analogous to the value of N, the knowledge of the Spiritual world is a solution of all secrets of the natural world.

I hope to go into this matter deeper in my second proposed book, " The Cheri-Bomb".

As Ever, Laurence M. Rash.