CHAPTER 5

<u>อนุกรม TRIGONOMETRY</u>

$$\sum_{n=1}^{n} \sin a_n \; ; \; a_n = \theta + (n-1)d$$

$$b_n = \sin a_n = \left(\frac{2\sin\frac{d}{2}}{2\sin\frac{d}{2}}\right)\cdot \sin a_n = \frac{2\sin a_n \cdot \sin\frac{d}{2}}{2\sin\frac{d}{2}}$$

$$= \frac{\cos\left(a_n - \frac{d}{2}\right) - \cos\left(a_n + \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$= \frac{\cos\left(a_n - \frac{d}{2}\right) - \cos\left(a_{n+1} - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$= f(n) - f(n+1) \; ; \; f(n) = \frac{\cos\left(a_n - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$S_n = f(1) - f(n+1) = \frac{\cos\left(a_1 - \frac{d}{2}\right) - \cos\left(a_{n+1} - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$= \frac{2\cdot\sin\left(\frac{\left(a_1 - \frac{d}{2}\right) + \left(a_{n+1} - \frac{d}{2}\right)}{2}\right)\cdot\sin\left(\frac{\left(a_{n+1} - \frac{d}{2}\right) - \left(a_1 - \frac{d}{2}\right)}{2}\right)}{2\sin\frac{d}{2}}$$

$$= \frac{\sin\left(\frac{a_1 + a_n}{2}\right)\cdot\sin\left(\frac{nd}{2}\right)}{\sin\frac{d}{2}} = \frac{\sin\left(\overline{\theta}\right)\cdot\sin\left(\frac{nd}{2}\right)}{\sin\frac{d}{2}}$$

$$\sum_{n=1}^{n}\left((-1)^{n+1}\sin a_n\right) \; ; \; a_n = \theta + (n-1)d$$

แนวคิด $\sum_{n=1}^{n}\left((-1)^{n+1}\sin a_n\right)$

$$= \sum_{n=1}^{n}\left(\sin\left(a_n + \frac{n-1}{2}\pi\right)\right)$$

$$= \sum_{n=1}^{n}\left(\sin b_n\right) \; ; \; b_n = \theta + (n-1)(d + \frac{\pi}{2})$$

$$= \frac{\sin\left(\frac{b_1 + b_n}{2}\right)\cdot\sin\left(\frac{nd_b}{2}\right)}{\sin\frac{d_b}{2}}$$

$$= \frac{\sin\left(\theta + \frac{(n-1)(2d+\pi)}{4}\right)\cdot \sin\left(\frac{n(2d+\pi)}{4}\right)}{\sin\frac{2d+\pi}{4}}$$

$$\sum_{n=1}^{n} \cos a_n \ ; \ a_n = \theta + (n-1)d$$

$$b_n = \cos a_n = \left(\frac{2\sin\frac{d}{2}}{2\sin\frac{d}{2}}\right)\cdot \cos a_n = \frac{2\cos a_n \cdot \sin\frac{d}{2}}{2\sin\frac{d}{2}}$$

$$= \frac{\sin\left(a_n + \frac{d}{2}\right) - \sin\left(a_n - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$= \frac{\sin\left(a_{n+1} - \frac{d}{2}\right) - \sin\left(a_n - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$= f(n+1) - f(n) \ ; \ f(n) = \frac{\sin\left(a_n - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$S_n = f(n+1) - f(1) = \frac{\sin\left(a_{n+1} - \frac{d}{2}\right) - \sin\left(a_1 - \frac{d}{2}\right)}{2\sin\frac{d}{2}}$$

$$= \frac{2\cdot\cos\left(\frac{\left(a_{n+1} - \frac{d}{2}\right) + \left(a_1 - \frac{d}{2}\right)}{2}\right)\cdot \sin\left(\frac{\left(a_{n+1} - \frac{d}{2}\right) - \left(a_1 - \frac{d}{2}\right)}{2}\right)}{2\sin\frac{d}{2}}$$

$$= \frac{\cos\left(\frac{a_1 + a_n}{2}\right)\cdot \sin\left(\frac{nd}{2}\right)}{\sin\frac{d}{2}} = \frac{\cos(\overline{\theta})\cdot \sin\left(\frac{nd}{2}\right)}{\sin\frac{d}{2}}$$

$$\sum_{n=1}^{n}\left((-1)^{n+1}\cos a_n\right) \ ; \ a_n = \theta + (n-1)d$$

แนวคิด $\sum_{n=1}^{n}\left((-1)^{n+1}\cos a_n\right)$

$$= \sum_{n=1}^{n}\left(\cos\left(a_n + (n-1)\pi\right)\right)$$

$$= \sum_{n=1}^{n}\left(\sin b_n\right) \ ; \ b_n = \theta + (n-1)(d+\pi)$$

$$= \frac{\cos\left(\frac{b_1 + b_n}{2}\right)\cdot \sin\left(\frac{nd_b}{2}\right)}{\sin\frac{d_b}{2}}$$

$$= \frac{\cos\left(\theta+\frac{(n-1)(d+\pi)}{2}\right)\cdot\sin\left(\frac{n(d+\pi)}{2}\right)}{\sin\frac{d+\pi}{2}}$$

Ex $\sum_{n=1}^{n}\sin^2 a_n \;;\; a_n = \theta+(n-1)d$

แนวคิด เนื่องจาก $2\sin^2\theta = 1-\text{coa}2\theta$

$$2\sum_{n=1}^{n}\sin^2 a_n = \sum_{n=1}^{n}\left(1-\cos 2a_n\right)$$

$$= n - \sum_{n=1}^{n}\cos b_n \;;\; b_n = 2a_n = 2\theta+(n-1)(2d)$$

$$= n - \frac{\cos\left(2\theta+(n-1)d\right)\cdot\sin(nd)}{\sin d}$$

$$\sum_{n=1}^{n}\sin^2 a_n = \frac{n}{2}-\frac{\cos\left(2\theta+(n-1)d\right)\cdot\sin(nd)}{2\sin d}$$

✍ $\sum_{n=1}^{n}\cos^3(2n-1)\alpha$

แนวคิด เนื่องจาก $4\cos^3\theta = 3\cos\theta+\cos 3\theta$

$$4\sum_{n=1}^{n}\cos^3(2n-1)\alpha = \sum_{n=1}^{n}\left(3\cos(2n-1)\alpha+\cos 3(2n-1)\alpha\right)$$

$$= 3\sum_{n=1}^{n}\cos(2n-1)\alpha+\sum_{n=1}^{n}\cos 3(2n-1)\alpha$$

$$= 3\cdot\frac{\sin\frac{2n\alpha}{2}}{\sin\frac{2\alpha}{2}}\cdot\cos\frac{\alpha+(2n-1)\alpha}{2} + \frac{\sin\frac{6n\alpha}{2}}{\sin\frac{6\alpha}{2}}\cdot\cos\frac{3\alpha+3(2n-1)\alpha}{2}$$

$$\sum_{n=1}^{n}\cos^3(2n-1)\alpha = \frac{3\sin n\alpha\cos n\alpha}{4\sin\alpha}+\frac{\sin 3n\alpha\cos 3n\alpha}{4\sin 3\alpha}$$

Ex $\sum_{n=1}^{n}\tan^{-1}\frac{x}{1+n(n+1)x^2}$

$$= \sum_{n=1}^{n}\left(\tan^{-1}(n+1)x-\tan^{-1}nx\right)$$

$$= \tan^{-1}(n+1)x - \tan^{-1}x$$

Ex $\sum_{n=1}^{n}\left(\frac{1}{2^{n-1}}\tan\frac{a}{2^{n-1}}\right)$

แนวคิด จาก $\tan a = \cot a - 2\cot 2a$

$$a_n = \frac{1}{2^{n-1}}\tan\frac{a}{2^{n-1}} = \frac{1}{2^{n-1}}\left(\cot\frac{a}{2^{n-1}}-2\cot\frac{2a}{2^{n-1}}\right)$$

$$= \frac{1}{2^{n-1}}\cot\frac{a}{2^{n-1}}-\frac{1}{2^{n-2}}\cot\frac{a}{2^{n-2}}$$

$$= f(n+1) - f(n) \;;\; f(n) = \frac{1}{2^{n-2}}\cot\frac{a}{2^{n-2}}$$

$$S_n = f(n+1) - f(1)$$

$$= \frac{1}{2^{n-1}}\cot\frac{a}{2^{n-1}} - 2\cot 2a$$