Innovation and Imitation at Various Stages of Development[1]

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Abstract

A simple model of imitation and innovation is developed to explain a complicated picture of relative productivity growth in different countries. The model makes difference between global and local innovations and does not assume that a country always imitates the most advanced technology. It is shown that there are three types of stationary states, where only imitation, only innovation or a mixed policy prevails. We demonstrate how one can find the stationary states and check their stability for a broad class of imitation-innovation cost functions.

Using World Bank statistical data for the period of 1980-1999, we reveal the dependence of innovation and imitation costs on GDP per capita measured in PPP and on an indicator of investment risk. An appropriate choice of two adjustment parameters of the model gives a possibility to generate trajectories of more than 80 countries and, for most of them, get qualitatively correct pictures of their movement. It turns out that three groups of countries behave differently, and there is a tendency to converge inside each group. Increase in institutional quality get countries out of underdevelopment traps, from the imitation area to a better steady state where local innovations and imitations are jointly used. All countries with high quality of institutions are moving toward the area where pure innovation policy prevails.

1 Introduction

A complicated picture of relative productivity growth in different countries is one of the main challenges for the economic science. European countries seem to be converging whereas most of developing economies keep their relative distance from the leaders unchanged or even fall behind. Only few East Asia economies were fortune to catch up, and several unique countries like China, or Vietnam demonstrates fast growth at the recent time. This paper aims to explain this picture in the framework of a simple model that takes into account the interaction between innovation and imitation processes.

Up to the end of eighties, innovation and imitation, two sides of the development process, were modeled separately. Segerstrom (1991) cites the only exception: a paper by Baldwin and Childs (1969) where firms can choose between innovating and imitating. Iwai (1984a,b) considered distributions of firms in an industry with respect to efficiency and suggested a model that describes evolution of the distribution curves. The model demonstrates a "dynamic equilibrium" between innovation and imitation processes. This approach, in principle, may be implemented to country distributions as well. It is developed in a number of papers by Henkin and Polterovich (see Polterovich, Henkin (1988) and Henkin, Polterovich (1999) for a survey). They assume that a firm moves to the neighboring efficiency level due to innovation and imitation of more advanced technologies, and the speed of the movement depends on the share of more advanced firms. The movement is described by a difference-differential equation that may be considered as an analogue of the famous Burgers partial differential equation. It is shown that innovation-imitation forces may split an isolated industry into several groups so that the solution looks like a combination of several isolated waves moving with different speeds.

Segerstrom (1991) develops a dynamic general equilibrium model and studies its steady state in which some firms devote resources to discovering new products and other firms copy them. Somewhat similar model is suggested in Barro and Sala-I-Martin (1995) to describe the interaction between innovation and imitation as an engine of economic growth. They show that follower countries tend to catch up to the leader because imitation is cheaper than innovation. Barro and Sala-I-Martin discuss also a number of related issues and papers devoted to technological diffusion, R&D races and leapfrogging. The theory of leapfrogging is developed in Brezis, Krugman, and Tsiddon (1993).

In a recent paper by Acemoglu, Aghion, and Zilibotti (2002), it is shown that countries at early stages of development use imitation strategy whereas more advanced economies switch to innovation-based policy. Relatively backward economies may switch out of investment-based strategy too soon or fail to switch at all. In the last case, they get into a non-convergence trap. The traps are more likely if the domestic credit market is imperfect. The authors receive also a number of other results that characterize policies for different stages of development.

Our model borrows a number of elements from Acemoglu, Aghion, and Zilibotti (2002). We concentrate on the innovation-imitation tradeoff and develop this part of their model.

Our approach is different from approaches used by other authors in several important aspects.

1. We do not assume, as it is done in other papers, that a country always imitates the most advanced technology. This assumption seems to be too strict. Every developing country experiences a lot of failures connected with attempts of borrowing the most recent achievements of the developed world. It is too often that modern techniques and technologies turn out to be incompatible with domestic culture, institutions, quality of human capital, or domestic technological structure. A rational policy admits borrowing of the past experience of the leaders. Imitation of a less advanced technology is cheaper and has more chances for a success.

2. We make difference between global and local innovations. The first ones may be borrowed by other countries whereas the second ones are country specific. It is quite plausible that the most part of R&D expenditures are spent for local innovations[1].

[1]In Pack (2001, p.114) the following passage from Ruttan (1997) is cited: "The technologies that are capable of becoming the most productive sources of growth are often location specific". Pack writes that

If one likes to use R&D expenditures as a measure for the innovation activity, one should not assume that all innovations produced are useful for other countries[2].

3. The larger is an innovation or an imitation, the less is the probability to produce it. Therefore, both policies exhibit decreasing rate of return. The tradeoff between innovation and imitation projects may result in producing both of them in positive proportions.

4. Costs of imitation and innovation in a country depend on its relative level of development. The level of development is defined as a ratio of the country productivity parameter to the productivity of the most advanced economy. A similar assumption is used in Acemoglu, Aghion, and Zilibotti (2002), where the costs per unit of the productivity increase are taken to be constant for imitation and linear for innovation. We consider quite general shape of cost functions. It is assumed, however, that the value of imitation cost function increases and the value of innovation cost function decreases when the economy approaches a leader. A similar assumption about the cost of imitation may be found in Barro and Sala-I-Martin (1995). The reason is that less advanced economies may borrow well-known and cheap technologies that may be even obsolete for advanced economies.

   The shape of dependence of unit innovation cost on development levels is more questionable. On the one hand, one could assume decreasing rate of return, on the other hand, accelerating effects of the technical progress occur. We analyze empirical data to demonstrate that unit innovation cost is probably less for most advanced economies.

5. We take into account the country's institutional quality that affects imitation and innovation cost functions. Since the institutional indicators are considered as ex-

"this view is widely shared among those who have done considerable research on the microeconomics of technology…".

[2]In Russia in 2001, R&D expenditures amounted to 100.5 billion rubles, whereas the value of exported technologies was only 19 billion rubles. (Russian Statistical year book, 2002. M.: Goskomstat, pp. 521–522). There were granted more than 16000 patents, and only 4 of those were exported.

ogenous, we study the behavior of the economy for a broad class of expenditures functions.

6. In most of investigations, steady states are studied only. However, to generate a picture that would resemble a real set of country trajectories, one has to consider transition paths. To do that, we are forced to simplify our model drastically. The model is quasi-static and generates trajectories as sequences of static equilibria[3]. We do not consider capital accumulation at all[4] and make use of many other simplifications as well.

It will be shown that in the simple model suggested below, there are three types of stationary states, where only imitation, only innovation or a mixed policy prevails. It is demonstrated how one can find the stationary states and check their stability for a broad class of imitation-innovation cost functions.

Using World Bank statistical data for the period of 1980-1999, we reveal dependence of innovation and imitation cost on GDP per capita measured in PPP and on indicator of investment risk. An appropriate choice of two adjustment parameters of the model gives a possibility to generate trajectories of more than 80 countries and, for most of them, get quantitatively correct pictures of their movement. Roughly speaking, three groups of countries behave differently. There is a tendency to converge within each group. Countries with low institutional quality have stable underdevelopment traps near the imitation area. Increase in the quality moves the steady state toward a better position and turns into a new stable steady state where local innovations and imitations are jointly used. Under further institutional improvements, a combined imitation-innovation underdevelopment trap disappears. All countries with high quality of institutions are moving toward the area where pure innovation policy prevails.

[3]This is a feature of the Acemoglu, Aghion, and Zilibotti's model as well.

[4]Many authors, who study endogenous growth models, use this simplification (see Aghion and Howitt, 1998, or Barro and Sala-I-Martin, 1995).

2 Description of the model

In this section, a simple model of imitation and innovation is introduced to explain the dynamics of relative productivity growth in different countries.

The basics of the model are the same as in Acemoglu, Aghion, and Zilibotti (2002a). We consider a multi-product economy evolving over time. Every period, final output y is produced from labor l and the continuum set of intermediate inputs x_ν ($\nu \in [0,1]$). Given A_ν, the productivity level achieved in sector ν, the final output is given by the following production function:

$$y = \frac{1}{\alpha} \int_0^1 (A_\nu l)^{1-\alpha} x_\nu^\alpha \, d\nu. \tag{1}$$

The output sector is competitive; it is assumed that the world price of output is 1 and the price of x_ν is p_ν. Then the inverse demand function for good x_ν is given by the marginal product of x_ν in (1):

$$p_\nu = \left(\frac{A_\nu l}{x_\nu} \right)^{1-\alpha}, \tag{2}$$

where $\alpha \in (0,1)$.

There are many firms in each sector ν but only one of them enjoys the full access to the most recent technology of producing x_ν: this leader firm can produce one unit of x_ν from one unit of output, whereas its rivals have to spend χ units of output for the same task ($1 < \chi < 1/\alpha$). Therefore, they fail to compete with the leader firm and produce nothing in equilibrium. However, the existence of the competitive fringe affects the price policy of the leader firm: the monopoly price $1/\alpha$ is not feasible and the leader has to set $p_\nu = \chi$. Thus, χ reflects the strength of monopoly power in high-technology industries.

In each sector ν, the leader firm lives for one period of time (unlike Acemoglu, Aghion, and Zilibotti (2002a), where firms live for two periods and form overlapping generations). Each leader firm starts with the average productivity level A. This level represents the cumulative knowledge achieved by the economy at the end of the previous period. In the beginning of the current period, the firm performs technological innovations and imitations, thus raising its productivity from A to A_ν, and then produces intermediate input at productivity level A_ν (it will be described later on, how this level is obtained).

The labor supply in the economy is assumed to be inelastic and equal to L. The profit of the leader firm π_ν is then given by

$$\pi_\nu = \delta L A_\nu; \tag{3}$$

where

$$\delta = (\chi - 1)\chi^{-\frac{1}{1-\alpha}} \quad \text{— increasing in } \chi.$$

A_{+1} is the average level of technology in the economy achieved at the end of the current period. We assume that next-period firms start their development from this productivity level, so A_{+1} plays the same role for them as A for current-period firms. Thereby, the spillover effect is exhibited: each firm slightly affects the productivity of its successors when developing its own technology or imitating foreign one, though this positive externality is not captured by the firm's profit.

The above considerations concern the domestic economy. There are also foreign countries with different productivity levels. Denote by $\bar{A}$, the (average) productivity level of the most developed economy at the end of the previous period. In addition to the domestic absolute productivity level A, let us consider the relative level $a = \dfrac{A}{\bar{A}}$ which measures the distance to the world technology frontier. It represents the position of the domestic technology among other ones.

Now let us describe the evolution of technologies. As we said, each firm performs imitation and/or innovation prior to production. Let b_1 and b_2 denote, respectively, the size of the imitation and innovation project. Each project may result in one of two outcomes, success or failure. If the imitation (innovation) was successful, firm's productivity rises at growth rate b_1 (b_2); otherwise, it remains the same. Thus, after both actions, the technology variable $A(\nu)$ is given by

$$A_\nu = (1 + \xi_1(\nu))(1 + \xi_2(\nu))A, \tag{4}$$

where $\xi_1(\nu)$ ($\xi_2(\nu)$) is a random variable equal to b_1 (b_2) in the case of successful imitation (innovation) by firm ν and 0 otherwise. Firms cannot imitate technologies which have not been developed anywhere in the world yet, so the size of the imitation project is subject to constraint

$$(1+b_1) \leq \frac{1}{a}, \tag{5}$$

where $a = \frac{A}{\bar{A}}$. The maximal productivity level $\bar{A}$ is generated by the leader economy and is not affected by innovations made by less advanced economies. Thus, we assume that the followers produce local innovations which are country-specific and cannot be imitated. We assume that the probabilities of success are, respectively, $\psi_1(b_1)$ and $\psi_2(b_2)$, where

$$\psi_1(b_1) = \frac{\mu_1}{\mu_1 + b_1};$$
$$\psi_2(b_2) = \frac{\mu_2}{\mu_2 + b_2}.$$

The postulated form of the probability function is the simplest one that satisfies natural boundary conditions: $\psi(0) = 1$, $\psi(\infty) = 0$. It will be shown below that μ_1 and μ_2 are maximal possible growth rates due to imitation and innovation, respectively.

The costs of imitation and innovation are assumed to be linear in b_i. Under the above assumptions, the expected profit of the firm is given by

$$E(\pi_\nu) = \delta L A_{+1} - c_1 b_1 - c_2 b_2, \tag{6}$$

where

$$A_{+1} = (1+\psi_1(b_1)b_1)(1+\psi_2(b_2)b_2)A \tag{7}$$

is just the next-period productivity level equal to the expectation of A_ν (note that despite all A_ν are stochastic, their average A_{+1} is non-random because the set of sectors is continual).

Per-unit costs c_1 and c_2 are assumed to depend on the absolute and relative average productivity level of our economy at the end of the previous period. Particularly, c_i are given by

$$c_i = \delta L A q_i(a), \tag{8}$$

where $q_1(a)$ is increasing and $q_2(a)$ is decreasing in a (moreover, both functions are assumed to be continuous in a). Thus, it gets more difficult to imitate and easier to innovate, as the domestic technology gets closer to the world technology frontier. These assumptions were discussed above and will be supported below by the empirical analysis.

Let us denote $\bar{q}_1 = q_1(1)$, $\bar{q}_2 = q_2(1)$. From now on, assume that $\bar{q}_2 < 1$. This assumption is needed to exclude the cases where no innovation is performed by any economy (see Figure 1 on page 13).

Thus, both forms of technological development are modeled in a similar way here. However, the opportunities for imitation and innovation change in different ways as a increases. In particular, when a is close to 1, there is almost nothing to imitate, whereas innovation is possible and its cost is low.

To summarize, the technology evolves as follows. At the beginning of the period, all leader firms start from the same productivity level A. Firms choose the sizes of their imitation and innovation projects which maximize their profits. Then random events are realized: success or failure of these projects (correspondingly, random variables A_ν are evaluated). Then production takes place and profits are earned. The next-period productivity level A_{+1} is the integral of A_ν over all $\nu \in [0, 1]$. All next-period firms start their projects from this level. They make their innovations and imitations, leaving productivity level A_{+2} to their successors, and so on.

3 Analysis of the model

In this section, imitation and innovation strategies in each country are obtained and the dynamics or relative growth is studied based on the model described above.

It is convenient for the further study to introduce alternative variables for imitation and innovation strategies. Let us denote $y_i = \psi_i(b_i)b_i$ and use y_i as independent variables instead of b_i. Then

$$b_i = \frac{\mu_i y_i}{\mu_i - y_i}. \tag{9}$$

Since any technology can be imitated only if it has been developed somewhere in the world (see (5)), y_1 is bounded from above:

$$y_1 \le \bar{y}_1(a) = \frac{1}{\dfrac{a}{1-a} + \dfrac{1}{\mu_1}} \tag{10}$$

Substituting (8) and (9) into (6), we can rewrite the optimization problem of the firm as

$$(1+y_1)(1+y_2) - \frac{\mu_1}{\mu_1 - y_1} q_1(a) y_1 - \frac{\mu_2}{\mu_2 - y_2} q_2(a) y_2 \to \max_{y_1, y_2} \tag{11}$$

subject to

$$y_1 \in [0, \bar{y}_1(a)]; \qquad y_2 \in [0, \mu_2). \tag{12}$$

If the technology imitated in the optimum is not the most advanced one ($y_1 < \bar{y}_1(a)$), then the first-order conditions are

$$\begin{aligned} y_1 &= \hat{y}_1(y_2) = \max\left(\mu_1\left(1 - \sqrt{\frac{q_1}{1+y_2}}\right), 0\right); \\ y_2 &= \hat{y}_2(y_1) = \max\left(\mu_2\left(1 - \sqrt{\frac{q_2}{1+y_1}}\right), 0\right), \end{aligned} \tag{13}$$

where $q_1 = q_1(a)$, $q_2 = q_2(a)$. Let (y_1, y_2) be the solution to (13). The first-order conditions yield the actual maximum of profits, if the following second-order condition hold:

$$(\mu_1 - y_1)(\mu_2 - y_2) < 4(1+y_1)(1+y_2). \tag{14}$$

In particular, it is sufficient for (14) to be valid that

$$\mu_1 \mu_2 < 4. \tag{15}$$

From now on, let us assume that (15) always holds.

Now let us determine y_1 and y_2. To begin with, suppose that constraint (10) is not binding. Then the the optimal solution (y_1, y_2) depends only on the position of point

$q = (q_1, q_2)$ (see Figure 1 on page 13: areas D_{00}, D_{0+}, D_{+0} and D_{++} separated with dotted lines). It is easy to check that if $q_1 \geq 1$ and $q_2 \geq 1$ ($q \in D_{00}$), then $y_1 = y_2 = 0$, i. e. the economy stagnates with no technological development. If $q_1 \geq 1 + \mu_2 \left(1 - \sqrt{q_2}\right)$ and $q_2 < 1$ ($q \in D_{0+}$), then $y_1 = 0$ and $y_2 = \mu_2 \left(1 - \sqrt{q_2}\right)$, i. e. there are only innovations. If $q_2 \geq 1 + \mu_1 \left(1 - \sqrt{q_1}\right)$ and $q_1 < 1$ ($q \in D_{+0}$), then $y_1 = \mu_1 \left(1 - \sqrt{q_1}\right)$ and $y_2 = 0$, i. e. there are only imitations. Otherwise (if $q \in D_{++}$), the optimum is interior ($0 < y_1 < \bar{y}_1(a), 0 < y_2 < \mu_2$). In this case, the analysis of equation (13) yields that y_i negatively depends on both q_1 and q_2. Thus, some kind of complementary effect takes place: if the per-unit cost of innovation gets down, then the imitation activity will rise even though the per-unit cost of imitation remains the same; an analogous effect for innovations is present too[5].

All of the above is true for any a only if constraint (10) is never binding, i. e. if

$$\hat{y}_1(\hat{y}_2(\bar{y}_1(a))) < \bar{y}_1(a) \tag{16}$$

for any a. If $a = 0$, then (16) holds automatically. If $a = 1$, then $\bar{y}_1(a) = 0$, so (10) is not binding, when

$$\bar{q}_1 \geq 1 + \mu_2 \left(1 - \sqrt{\bar{q}_2}\right). \tag{17}$$

In fact, (17) just means that $\bar{q} = (\bar{q}_1, \bar{q}_2) \in D_{0+}$ (see Figure 1). As follows from the above considerations, if (17) does not hold, then there must exist $\bar{a} \in [0, 1]$, such that (16) holds for $a < \bar{a}$ and does not hold for $a \geq \bar{a}$ for all a sufficiently close to $\bar{a}$. This $\bar{a}$ can be determined from the following equation:

$$\hat{y}_1(\hat{y}_2(\bar{y}_1(a))) = \bar{y}_1(a). \tag{18}$$

Is it true that the solution to (18) does not exist, when (17) holds, and is unique, otherwise? Generally, one cannot be sure about this, however, it is a typical situation. In particular, it can be shown that the above statement is true for low μ_1 and μ_2 (for example, this is the case, when the firm's time horizon is short, say, one year).

[5] These effects are subtle, if μ_1 and μ_2 are low.

If (17) does not hold ($\bar{q} \in D_{++}$), then constraint (10) is binding for sufficiently high a. In this case, $y_1 = \bar{y}_1(a)$ and $y_2 = \hat{y}_2(\bar{y}_1(a))$, i. e. the most advanced technology is imitated.

It is convenient in this case to consider instead of $q_1(\cdot)$, the "adjusted" per-unit cost function

$$\tilde{q}_1(a) = \begin{cases} q_1(a), & \text{if (16) holds;} \\ \text{solution to } \hat{y}_1(\hat{y}_2(\bar{y}_1(a))) = \bar{y}_1(a) \text{ with respect to } q_1, & \text{otherwise,} \end{cases} \tag{19}$$

where q_1 is treated as an independent variable in the lower equation. Obviously, if (16) does not hold, then $\tilde{q}_1(a) \geq q_1(a)$. In this case, the adjusted per-unit cost $\tilde{q}_1(a)$ is given by

$$\tilde{q}_1(a) = \mu_1^2 a^2 \frac{1 + \mu_2 \max\left(1 - \sqrt{\dfrac{1 - a + \mu_1 a}{1 - a + \mu_1} q_2(a)}, 0\right)}{(1 - a + \mu_1 a)^2}. \tag{20}$$

In particular, if (17) does not hold, then $\tilde{q}(1) = (\tilde{q}_1(1), q_2(1))$ lies at the left boundary of D_{0+} (see Figure 1b).

If $q(a)$ is replaced with $\tilde{q}(a) = (\tilde{q}_1(a), q_2(a))$, the solution to the profit maximization problem remains the same but constraint (17) is now never binding, so the solution is always determined by the analog of (13).

Now let us study the dynamics of the system. The relationship between the new productivity level A_{+1} and the old one A is given by (7). Thus, the absolute productivity level of any economy is growing over time or, at least, staying the same. However, the relative productivity level a may fall because some economies may grow slower than the world technology frontier moves. Suppose that there is no leapfrogging, i. e. the highest average productivity level in the world at the end of the current period is reached by the same country as that one period ago[6]. Then the evolution of a is determined by

$$a_{+1}(a) = \frac{A_{+1}}{\bar{A}_{+1}} = \frac{(1 + y_1)(1 + y_2)}{1 + \bar{g}} a, \tag{21}$$

where $\bar{g}$ is the growth rate of the most advanced economy:

$$\bar{g} = \frac{\bar{A}_{+1}}{\bar{A}} - 1. \tag{22}$$

[6]The case of leapfrogging will be considered later on, see page 16

The most advanced economy is unable to imitate technologies, it can only innovate, so $y_1 = 0$ in this economy. Hence,

$$\bar{g} = \hat{y}_2(0) = \mu_2 \left(1 - \sqrt{\bar{q}_2}\right). \tag{23}$$

The evolution equation (21) can be treated as an autonomous difference equation: its right-hand side does not depend neither on time, nor on individual characteristics of countries other than the phase variable a. Hence, in order to predict the process of evolution and the eventual state of the system, one just has to know, for which a the difference $a_{+1}(a) - a$ is positive and for which a it is negative.

Let us define the *cost curve* within plain (q_1, q_2) as the set of pairs $(\tilde{q}_1(a), q_2(a))$ for $a \in [0, 1]$. Denote $\varphi(q_1) = q_2(\tilde{q}_1^{-1}(q_1))$. Then the cost curve is fully determined by $\varphi(\cdot)$ and the edge values $\tilde{q}_1(0), \tilde{q}_1(1)$: it consists of pairs $(q_1, \varphi(q_1))$, where $q_1 \in [\tilde{q}_1(0), \tilde{q}_1(1)]$.

Suppose that $\bar{q}_2 = \varphi(\tilde{q}_1(1))$ is given. Let us define the *steady-state curve* as the set of potential steady-state pairs (q_1, q_2), i. e. pairs for which $a_{+1}(a) = a$, where $a = \tilde{q}_1^{-1}(q_1)$ (due to (21) and (23), this definition is correct).

Within D_{0+}, the steady-state curve is a horizontal line with $q_2 = \bar{q}_2$. So, if $a < 1$ and $\tilde{q}(a) \in D_{0+}$ (this may be the case for relatively high a and only when (17) holds), then there is no imitation the distance to frontier will definitely increase (see Figure 1a).

Within D_{+0}, the steady-state curve is a vertical line with $q_1 = \breve{q}_1$, where

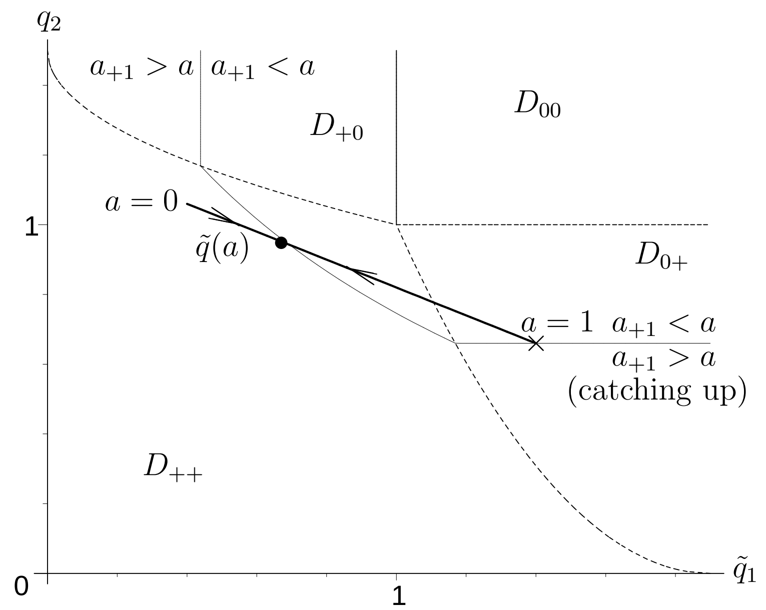
$$\breve{q}_1 = \left(1 - \frac{\mu_2}{\mu_1}\left(1 - \sqrt{\bar{q}_2}\right)\right)^2, \tag{24}$$

so the direction of the evolution depends only on the cost of imitation in this case.

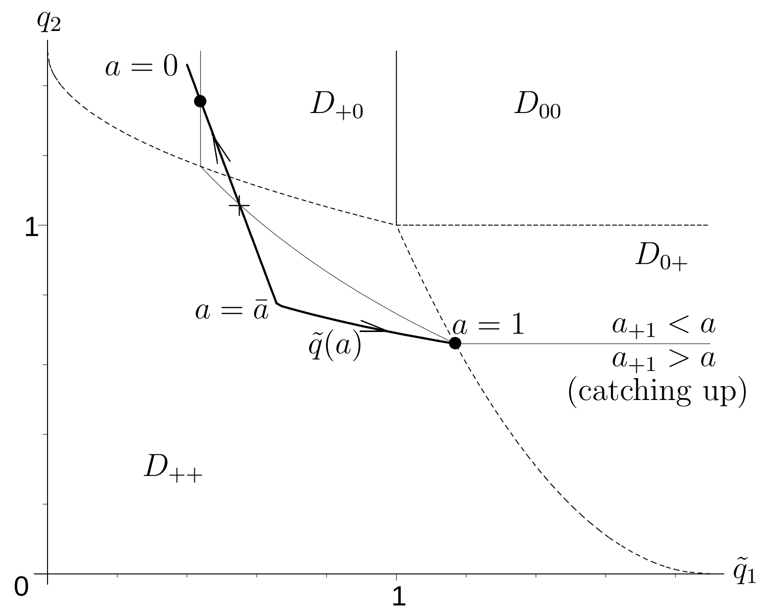
Within D_{++}, the steady-state curve is determined by the following equation:

$$(1+y_1)(1+y_2) = 1+\bar{g}, \tag{25}$$

where pair (y_1, y_2) can be found from (13), with $\tilde{q}_1$ in place of q_1, and $\bar{g}$ is given by (22). Since both y_1 and y_2 are decreasing in q_1 and q_2, equation (25) determines a downward-sloping curve in plain (q_1, q_2).



a) Constraint (10) is never binding.



b) Constraint (10) is binding for some a.

D_{++}: $y_1 > 0$, $y_2 > 0$ D_{0+}: $y_1 = 0$, $y_2 > 0$ • stable stationary point

D_{+0}: $y_1 > 0$, $y_2 = 0$ D_{00}: $y_1 = 0$, $y_2 = 0$ × unstable stationary point

Figure 1: Cost curve $\tilde{q}\,(a)$, catching-up area and steady states.

The entire steady-state curve is depicted in Figure 1a,b. The position of this curve depends only on $\bar{q}_2$ and does not depend on the position of the rest of the cost curve.

Now, to qualitatively analyze the dynamics, it is sufficient to know, how the cost curve is positioned about the steady-state curve. The relative productivity level of the economy rises after the period if and only if point $q(a)$ lies below and/or to the left from the steady-state curve (let us call the corresponding zone "catching-up area"). Steady states correspond to intersection points of the cost curve with the steady-state curve. If $\varphi(q_1)$ gets out of the catching-up area as q_1 goes up (in other words, the slope of the cost curve is flatter that the slope of the steady-state curve), then the corresponding stationary point is stable; otherwise, it is unstable. For example, in Figure 1a, there is one stable steady state with $a < 1$ and one unstable[7], with $a = 1$. In Figure 1b, there are two steady states, with $a = 1$ and with low a; there is also an unstable state with intermediate a which is a boundary between the zones of attraction of the stable states.

In particular, an important question is whether the close pursuers of the leader economy will get even closer to the technology frontier or the leader will gradually move away from them. Obviously, $a_{+1}(1) = 1$ in the case of no leapfrogging, so the pursuers will reduce the gap if and only if

$$\left.\frac{da_{+1}}{da}\right|_{a=1} < 1. \tag{26}$$

It is clear that some imitation activity is needed to reduce the gap, so condition (26) will definitely not hold if $y_1 = 0$ for a close to 1, i. e. if (17) holds as a strict inequality (see Figure 1b). Otherwise, $y_1 = \bar{y}_1(a)$ and $y_2 = \hat{y}_2(\bar{y}_1(a))$ in a close neighborhood of 1, so the left-hand side of (26) can be easily calculated using (10) and (13). We obtain the following condition:

$$\bar{e}_2 < 2\frac{1 + 1/\mu_2}{\sqrt{\bar{q}_2}} - 1, \tag{27}$$

[7] The number of stationary points in dynamic systems is usually odd. Here the number is even because one of the points lies at the boundary of the phase space, which is $[0, 1]$. This point may be treated as two coincident stationary points, one stable and one unstable.

where $\bar{e}_2$ is the elasticity of innovation cost in a around the leader:

$$\bar{e}_2 = -\frac{q_2'(1)}{\bar{q}_2}.$$

In particular, (27) holds for sure, if $\bar{e}_2 < 1$. The intuitive sense of condition (27) is clear: if the innovation efficiency gap between the leader and its pursuers is not very wide, then the pursuers have a chance to catch up the leader combining innovation and imitation activity (though it is easier for the leader to innovate, it cannot imitate at all).

All of the above is true only if the relative productivity level a is the only parameter in which countries may vary. However, the growth opportunities of countries actually depend on many other factors. For example, suppose that cost q_i depends on the level of institutional development I: $q_i = q_i(a, I)$. In this case, our dynamical system is not autonomous and countries may overtake and surpass each other due to differences in I. However, the principles of our analysis are applicable for this more general case as well: as far as the leader country remains the same, the position of the steady-state curve does not changes, so all what we need is to draw cost curves for the values of I we are interested in and look at points where they intersect the steady-state curve (see Figure 2). Of course, analysis gets much more complicated in the presence of institutional changes, especially when they are endogenous to some extent.

Now let us study how innovation and imitation activities depend on the relative productivity level. Since imitation becomes more difficult and innovation easier as a increases, it seems that y_1 is likely to fall and y_2 is likely to rise as a gets higher. However, this is only a trend, not a precise fact. Moreover, the above-mentioned complementary effect may bring about some ambiguity: for example, a rises $\Rightarrow$ y_1 falls $\Rightarrow$ y_2 must somewhat fall too.

Implicit differentiation of (13) yields that the direction of the effect of an increase in a depends on e_{21}, the elasticity of substitution between imitation and innovation (or, geometrically, on the slope of the cost curve plotted in the log scale):

$$\begin{aligned} \frac{\partial y_1}{\partial a} > 0, \quad \text{iff } e_{21} > \frac{2(1+y_2)}{\mu_2 - y_2}; \\ \frac{\partial y_2}{\partial a} > 0, \quad \text{iff } e_{21} > \frac{\mu_1 - y_1}{2(1+y_1)}, \end{aligned} \tag{28}$$

where

$$e_{21} = -\frac{\tilde{q}_1(a) q_2'(a)}{q_2(a) \tilde{q}_1'(a)} = -\varphi'(\tilde{q}_1) \frac{\tilde{q}_1(a)}{q_2(a)}.$$

One can see from (28) that the higher e_{21}, the more probable a positive impact of an increase in a on innovation and imitation activities. This is not surprising: the steeper the slope of the cost curve is, the closer point $q(a)$ gets to the origin which means that both forms of technological development are likely to be enhanced. The "intuitively expected" outcome (more innovation, less imitation) takes place only for intermediate levels of e_{21}:

$$\frac{\mu_1 - y_1}{2(1+y_1)} < e_{21} < \frac{2(1+y_2)}{\mu_2 - y_2} \tag{29}$$

(as follows from (15), the left-hand side of (29) is always less than the right-hand side). A simple sufficient condition for (29) is

$$\frac{\mu_1}{2} < e_{21} < \frac{2}{\mu_2} \tag{30}$$

Now let us turn to the issue of leapfrogging. All of the above is true only when the right-hand side of (21) does not exceed 1 for all countries involved in the model, otherwise the leading position will be occupied by different countries in different periods (which is called "leapfrogging"). If we consider only close pursuers of the current leader as reasonable candidates for a new leader[8], then it is necessary for leapfrogging to be possible that

$$\left. \frac{\partial a_{+1}^{nlf}}{\partial a} \right|_{a=1} < 0, \tag{31}$$

where a_{+1}^{nlf} is just a_{+1} under no leapfrogging (see the right-hand side of (21)). Condition (31) is equivalent to the following inequality:

$$\bar{e}_2 < \frac{2}{\sqrt{\bar{q}_2}} - 1. \tag{32}$$

[8]Of course, one could construct an example of leapfrogging in which close pursuers cannot surpass the leader. However, this situation seems to be too exotic.

In particular, if $\bar{e}_2 < 1$, then leapfrogging is definitely possible. Leapfrogging will actually take place if some countries have their a sufficiently close to 1.

Sequential change of one leader with another may be generally a very complicated, even chaotic process. To illustrate the essence of leapfrogging, consider a process in which two countries, S and T each time replace one another at the leadership position: initially, country S has $a_S = 1$ and T has $a_T = \bar{a} < 1$; then they change their roles: $a_T = 1$, $a_S = \bar{a}$, and so on. This process is stationary if

$$a_{+1}^{nlf}(\bar{a}) = \frac{1}{\bar{a}}. \tag{33}$$

The growth rate of the world technology frontier is now given by

$$\bar{g} = \frac{1 + \mu_2 \left(1 - \sqrt{\bar{q}_2}\right)}{\bar{a}} - 1.$$

Thus, the frontier moves faster than in the case of no leapfrogging. This results in shifting the steady-state curve down and to the left, comparing to Figure (1b). Thus, while the two leaders help each other grow faster, the other countries have less opportunities for approaching the frontier. In particular, all stable steady states get lower and all unstable ones get higher (so, traps get wider).

4 Empirical adjustment of the model

In this section, some empirical considerations will be given about the model considered above. The model is far from being universal, it takes into account only a few factors. So, we do not intend to precisely test the model. All we are going to do is to choose its parameters so as to adjust it to the existing data.

The source of the data is the World Development Report (2001). The structure of the data is the following:

Y_t – GDP per capita, PPP (current international \$) in year $1900+t$,
$t = 80, \ldots, 99$, 87 countries;

I – quality of institutions (aggregate index of investment risks, 2000–2001),
$I$is higher for lower risks; 87 countries;

C_1 – spending on imitation activity (net royalty payments, % of GDP),
average over 1980–1999, 27 countries;

C_2 – spending on innovation activity (R&D), % of GDP,
average over 1980–1999, 27 countries.

Based on these data, we construct other variables useful for our analysis:

$a_t = \dfrac{Y_t}{Y_t[\text{USA}]}$ – proxy for relative productivity level (for each country);

$a^{\text{av}} = (a_{80} \ldots a_{99})^{1/20}$ – geometrical average over all a_t (for each country);

$g = \left(\dfrac{Y_{99}}{Y_{80}}\right)^{1/19} - 1$ – average growth rate of GDP (for each country);

$g_a = \left(\dfrac{a_{99}}{a_{80}}\right)^{1/19} - 1$ – average growth rate of a_t (for each country).

We introduce a_t in such a way because USA have been a permanent leader in Y_t for all the period 1980–1999 (except for a few fluctuations).

The main difficulty in our analysis is that we do not know the values of q_1 and q_2 and observe C_1 and C_2 instead. Thus, we need a method to calculate q_1 and q_2. Our approach is the following: we assume that expenditures C_i are proportional to $\dfrac{\mu_i}{\mu_i - y_i} q_i y_i$ and based on this assumption, construct proxies for y_1 and y_2, the shares of imitation and innovation in the total growth rate g. Provided that y_1 and y_2 are positive, q_1 and q_2 can be easily obtained due to the first-order conditions (13):

$$
\begin{aligned}
q_1 &= \frac{(\mu_1 - y_1)^2 (1 + y_2)}{\mu_1^2}; \\
q_2 &= \frac{(\mu_2 - y_2)^2 (1 + y_1)}{\mu_2^2}.
\end{aligned}
\tag{34}
$$

Let us denote

$$\gamma = \frac{\mu_2 C_2}{\mu_1 C_1}.$$

Using (34), one can obtain:

$$\gamma = \frac{(\mu_2 - y_2)(1 + y_1)y_2}{(\mu_1 - y_1)(1 + y_2)y_1} \tag{35}$$

Since all values of g in the sample are small (actually, lower than 0.12), one can expect (see formula (7)) that y_1 and y_2 are small too. So, in order to simplify calculations, let us linearize the model within a neighborhood of $(y_1, y_2) = (0, 0)$ (the error occurring due to this simplification is expected to have a very small share in the total estimation error). In particular, multipliers $(1+y_i)$ in (35) can be neglected[9]. Thus, (35) can be approximately rewritten as

$$\gamma = \frac{(\mu_2 - y_2)y_2}{(\mu_1 - y_1)y_1}, \tag{36}$$

where y_1 and y_2 are tied with an approximate equality

$$y_1 + y_2 = g. \tag{37}$$

Let us set μ_1 and μ_2 at some level. Solving (36)–(37) for (y_1, y_2) and adopting constraints (12), we obtain the following proxies for y_1 and y_2:

$$\begin{aligned} y_1 &= \max\left(\min\left(\tilde{y}_1, \bar{y}_1(a^{\mathrm{av}})\right), 0\right); \\ y_2 &= g - y_1, \end{aligned} \tag{38}$$

where

$$\tilde{y}_1 = \frac{\gamma\mu_1 + \mu_2 - 2g - \sqrt{(\gamma\mu_1 + \mu_2)^2 - 4\gamma g(\mu_1 + \mu_2 - g)}}{2(\gamma - 1)} \tag{39}$$

(it turns out that $\gamma > 1$ for each country from the sample, if μ_1 is not much higher than μ_2).

As far as y_1 and y_2 are calculated, q_1 and q_2 can be obtained from (34).

Our next task is to estimate q_1 and q_2 as functions of a and institution variable I, taking μ_1 and μ_2 as given. We have chosen the linear representation for the cost functions:

$$\begin{aligned} q_1(a, I) &= \beta_{a1}a + \beta_{I1}I + \beta_1; \\ q_2(a, I) &= \beta_{a2}a + \beta_{I2}I + \beta_2, \end{aligned} \tag{40}$$

[9]Note that y_i in $(\mu_i - y_i)$ cannot be neglected because μ_i, the maximal possible growth rate, may be small as well as y_i.

For a given pair (μ_1, μ_2), parameters of functions (40) can be estimated using OLS (with a^{av} staying for a in the regressions). Thus, we have all parameters of the model.

Now we can study the evolution of the model world and compare it to that of the real world.

As before, let us deal with the linearized model. The first-order conditions (13) can be approximately rewritten for small y_1 and y_2 as

$$\begin{aligned} y_1 &= \min\left(\max\left(\mu_1\left(1-\sqrt{q_1(a,I)}\right)+\frac{\mu_1 y_2\sqrt{q_1}}{2},0\right),\bar{y}_1(a)\right); \\ y_2 &= \max\left(\mu_2\left(1-\sqrt{q_2(a,I)}\right)+\frac{\mu_2 y_1\sqrt{q_2}}{2},0\right). \end{aligned} \tag{41}$$

Here constraints (12) are taken into account. System (41) can be easily solved for (y_1, y_2), so we obtain functions $\check{y}_1(a,I)$ and $\check{y}_2(a,I)$ (these are the predicted values of the imitation and innovation growth rates; they do not coincide with our proxies for y_1 and y_2). Now one can easily construct a recurrent formula for a_{+1} as a function of a and I:

$$a_{+1}(a,I) = (1+\check{y}_1(a,I)+\check{y}_2(a,I)-\check{y}_2(1,I[\mathrm{USA}]))a \tag{42}$$

(formula (42) is just the linearization for (21)). Equation (42) determines the evolution of the model. In order to check, whether the model is consistent with the reality, let us try to predict the relative productivity level of countries in 1999, given that in 1980:

$$\breve{a}_{99} = a_{+1}(a_{+1}(\ldots a_{+1}(a_{80},I)\ldots,I),I) \qquad \text{(19 times)}$$

Now we can choose μ_1 and μ_2 so as to achieve the best prediction. But how to measure the quality of prediction? There are several reasonable criteria. Let us list some of them, with the optimal μ_1 and μ_2 (according to the corresponding criterion) in parentheses.

Minimal sum of squares ($\mu_1 = 0.073$, $\mu_2 = 0.098$): the sum of squares of error $a_{99}-\breve{a}_{99}$ is minimized. This criterion is sensitive to outliers.

Coincidence of directions ($\mu_1 = 0.081$, $\mu_2 = 0.125$): the predicted direction of change in a during the period 1980–1999 must coincide with the actual one for as many countries as possible. This criterion does not take into account the speed of changes but it is robust to outliers.

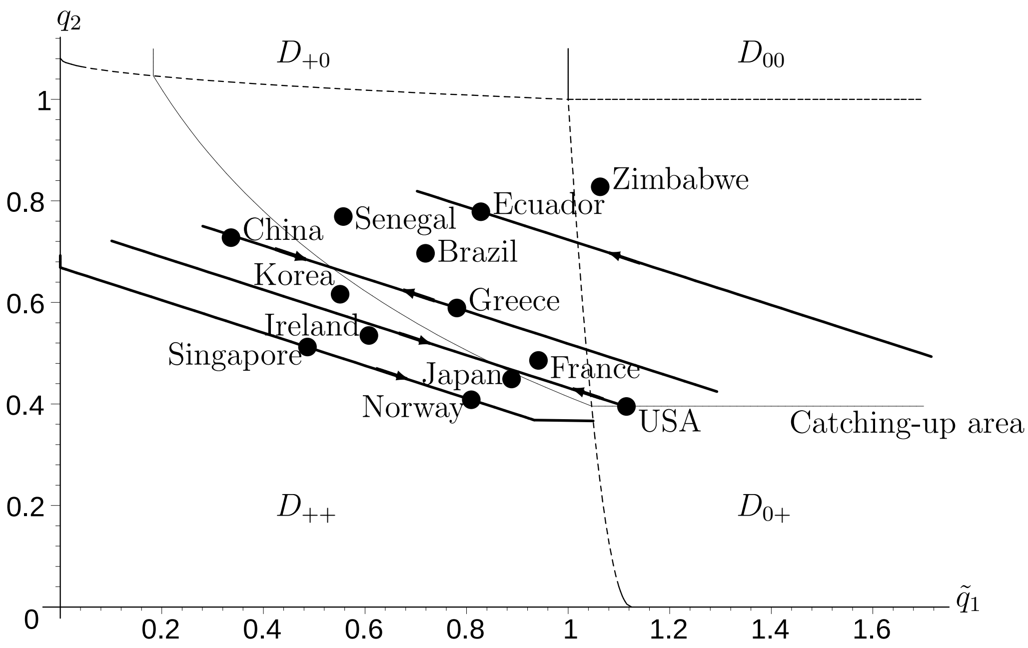


Figure 2: Cost curves for various institutional levels.

Coincidence of distributions ($\mu_1 = 0.071$, $\mu_2 = 0.069$): parameters are chosen so that the density function of distribution of $\ln \breve{a}_{99}$ be close to that of $\ln a_{99}$ (according to the least squares criterion). This criterion is useful for dealing with distribution densities rather than detailed cross-country data.

For example, suppose that μ_1 and μ_2 are chosen according to the second criterion ($\mu_1 = 0.081$, $\mu_2 = 0.125$). Then the estimated cost functions take the form

$$\begin{aligned} q_1(a, I) &= \underset{(4.46)}{1.012}a - \underset{(-2.32)}{0.0206}I + \underset{(2.99)}{1.790}; \\ q_2(a, I) &= \underset{(-1.43)}{-0.325}a - \underset{(-0.38)}{0.0034}I + \underset{(1.66)}{0.997} \end{aligned} \tag{43}$$

(t-statistics are given below the coefficients, in parentheses). The best quality of the directional prediction is 83% (the prediction is incorrect for 15 countries out of 86, not including USA).

For comparison, consider the regressions with I not included:

$$\begin{aligned} q_1(a, I) &= \underset{(3.75)}{0.624}a + \underset{(4.65)}{0.413}; \\ q_2(a, I) &= \underset{(-2.57)}{-0.325}a + \underset{(9.54)}{0.997}. \end{aligned} \tag{44}$$

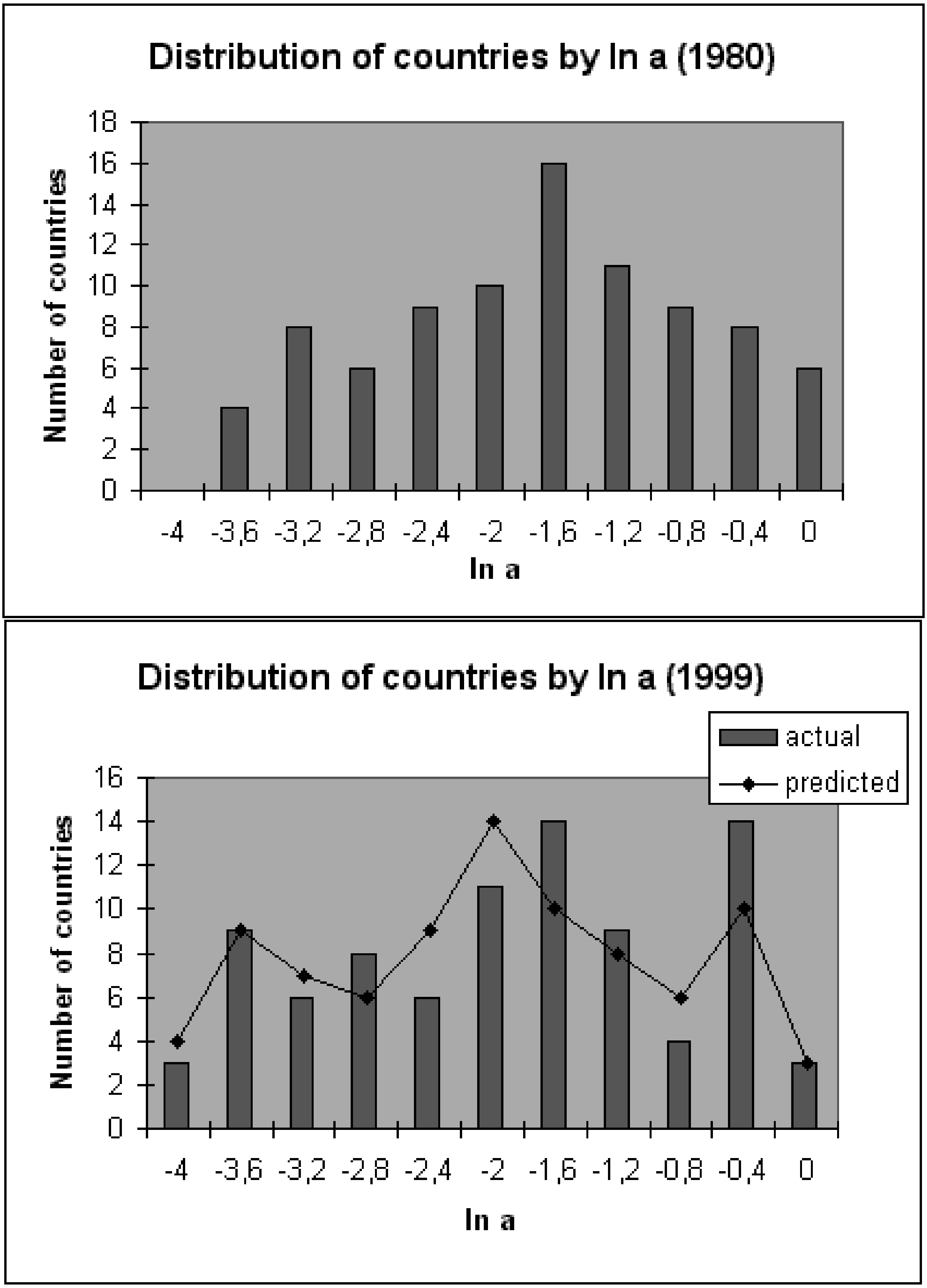


Figure 3: Distribution of countries with respect to $\ln a$

The directional prediction is only 66%.

Note that the signs of coefficients for a in (43) and (44) coincide with those postulated in the theoretical model. Another conclusion from the regressions is about the importance of the institutional variable for estimating the cost functions. However, the analysis of t-statistics shows that I significantly improves the precision of estimation only for the cost function of imitation, whereas its impact on the cost of innovation is insignificant. One of possible explanations is that the data we use concern the expenditures on local innovations. Only a small part of these R&D expenditures is materialized in large investment projects, so this activity does not require good investment climate.

Figure 2 depicts typical cases of positioning the estimated cost curve (43) with respect to the steady-state curve. If I is low (e. g., if the country has the same institutional index as Ecuador), then all of the cost curve lies out of the catching-up area, so the country will gradually fall to the lowest possible productivity level ($a = 0$). This is a stable stationary point (trap) and the corresponding (q_1, q_2) are near to the imitation area D_{+0}.

For intermediate I (e. g., Greece), the zero steady state ceases to be stable and a new stable steady state occurs with positive a. However, the country cannot catch up the leader.

If the country has the same quality of institutions as USA, then the picture becomes similar to Figure 1a: an additional unstable steady state $a = 1$ occurs.

For a little higher I (e. g., Japan), we would have the cost curve positioned a little lower than the curve for USA, so three stationary points would occur in this case: two stable ($a = 1$ and $a < 1$) and one unstable between them. If the country starts above the unstable steady state, then it will eventually catch up the leader; otherwise, it will get to the trap with $a < 1$.

Finally, for the highest I (Singapore, Norway), all of the cost curve lies in the catching-up area, so the country will eventually catch up the leader, no matter where it has started from.

The evolution of the distribution of $\ln a$ is depicted in Figure 3 (μ_1 and μ_2 are chosen so as to better predict the distribution density: $\mu_1 = 0.071$, $\mu_2 = 0.069$; the direction of change in a is predicted correctly for 73% countries in this case). One can see that in 1980, the distribution looks like the normal (unimodal) one, whereas in 1999, it has at least three local peaks. The above theoretical analysis suggests that these peaks may correspond to stable fixed points of $a_{+1}(a, I)$ for the most typical levels of I. However, another situation is possible: two "waves" of countries moving in the opposite directions may approach each other and "interfere". In particular, there is a group of intensively growing countries with relatively high quality of institutions: Norway, Ireland, Japan, Sweden, Finland, Hong Kong, Singapore. These countries have a real opportunity to approach the leader (USA). However, within the same peak, there are some other countries with weaker institutions and decreasing a: Belgium, Austria, France, Italy. They are moving away from the frontier.

5 Conclusion

The results described above makes plausible that the convergence problem, a central problem of the economic growth theory, may be better understood in the framework of an imitation-innovation theory if quality of institutions is taken into account. However, our model has to be developed to get a richer picture that would be closer to reality.

In a recent empirical study, Polterovich and Popov (2003, Stages of Development and Economic Growth. Manuscript) find that "imports of technology is unambiguously good for poor countries, but for middle and high income countries it should be supplemented with own research and development activity in order to have a positive impact on performance." On the one hand, this finding supports our model that demonstrates why innovation and imitation may be complementary. On the other hand, however, there is a divergence: the most intensive innovators, including USA, imitate a lot. Thus, one has to take into account that a part of followers' innovations is not local and may be borrowed. This part increases when the country level of development gets higher.

It is quite plausible that the cost of imitation depends not only on the distance to frontier but also on the position of a follower among other countries. This line of generalization leads us to a class of models that were started to study by Henkin and Polterovich (1999).

Institutional qualities of a number of economies like Canada or Norway are very close if not better to that of USA. Why does the USA keep its leadership so many years? To answer this question one may try to take into account capital accumulation, consumption, trade, foreign direct investment, and labor mobility. Another point is that followers pay to the leader for the right of imitation.

The role of international assistance is one of the most intrigue questions. It is well known that high inequality is harmful for growth. One could ask how the total world wealth might be redistributed to increase growth rates and consumption levels of all countries. This is an important topic for future research.

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