3986.3137—A NUMBER TO REMEMBER

If you're a nerdy Diaskhismaniac who's into numbers, here's a number to remember:

3986.313713864835 (or, for all practical purposes, simply 3986.3137)

Why this number?

Because it's the key to converting
fractions (representing the frequency ratios of musical intervals)
into
cents (a measurement of interval sizes—100 cents = 1 equal semitone, 1200 cents = 1 octave)

Here's an example of how it works.

An octave is the doubling of a frequency: 2/1, or simply 2.
The [base 10] logarithm of 2 = 0.30103
0.30103 x 3986.3137 = 1200 cents (1 octave)

That was pretty easy, wasn't it?

Let's do it with a harmonically pure perfect fifth. Its size is defined by partials 2 and 3 of the harmonic series.
Thus the frequency ratio of a pure perfect fifth is 3/2, or 1.5/1, or simply 1.5.
The logarithm of 1.5 = 0.17609
0.17609 x 3986.3137 = 701.955 cents

Here's another example, with an intriguing result.
The frequency ratio of a harmonically pure major third is 5/4, or 1.25.
The logarithm of 1.25 = 0.09691
0.09691 x 3986.3137 = 386.3137 cents
Here we discover this intriguing fact:
The conversion factor 3986.3137 = 3600 + 386.3137,
that is, 3 octaves plus a pure major third!
Why is this? I really don't know! Can you solve this mystery?

You can, of course, reverse the conversion process.

We know, by definition, that an equal-tempered perfect fifth is 700 cents, not 701.955 cents.
700 / 3986.3137 = 0.1756
This is the logarithm of 1.4983, slightly smaller than the 1.5 pure fifth!

Now let's compare the frequencies of a violin E string tuned
pure: 440Hz x 1.5 = 660Hz
tempered: 440Hz x 1.4983 = 659.252Hz
The slightly out-of-tune E would "beat" about twice a second, since its octave E harmonic would not match the E harmonic of the A string. The tiny difference of tuning, 1.955 cents (a skhisma), is at the limit of most people's ability to distinguish pitch differences.

However, equal temperament offers more dramatic departures from purely tuned intervals. Let's look at the major third. The pure major third, 5/4, sounds consonant because the 5th partial of the lower note matches the 4th partial of the upper note. So, for example, the pure major third above A440 is C#550, and both notes share a harmonic C#2200.

But doing the math for a tempered major third, we find that 400 cents / 3986.3137 = 0.10034, which is the log of 1.2599, which gives us a C#554.365. At first glance this would appear to beat more than 4 times a second against a C#550. However, the major third A440 and C#554.365 produces beats 4 times faster (more than 16 beats/sec.) between the C# harmonics two octaves higher that do not synchronize in this harmonically out-of-tune interval!

Another fact to consider is that two notes sounding at the same time produce the sensation of additional tones, called difference tones, whose frequencies, in simple terms, are the difference between the frequencies of the two notes actually being played. So, for example, C#550 and A440 would produce a difference tone A110. But what difference tone would be produced by C#554 and A440? A noticeably sharp A114—quite obviously not in tune with A440! This tempered third would sound rough due to both its clashing C# harmonics and its rumblingly out-of-tune difference tone!

Since any obsessively inquisitive Diaskhismaniac can spend hours exploring and discovering other fascinating relationships using the tool of our mysterious conversion factor, I'll stop here and let you go off on your own exciting quests. Let me remind you that the mystery of why 3986.3137 is the number of cents in 3 octaves plus a pure major third awaits a solution, one that could have cosmic consequences, perhaps even heralding a paradigm shift in human consciousness!! (But don't get your hopes up too high.)

SUGGESTIONS FOR FURTHER READING

The true Diaskhismaniac may wish to read this:
Michael Kimber, *Intonation: What Your Teacher Never Told You!*
http://m_kimber.tripod.com/intonation_untold.pdf
and perhaps this:
Michael Kimber, *Playing Melodically and Harmonically in Tune*
http://m_kimber.tripod.com/har_in_tune.pdf
and at least parts of this:
Michael Kimber, *Intonation Variables in the Performance of Twelve-Tone Music*
http://m_kimber.tripod.com/12-tone_intonation.pdf

And any Diaskhismaniac who reads German should become familiar with the main points in this book:
Christine Heman, *Intonation auf Streichinstrumenten: melodisches und harmonisches Hören* (Bärenreiter, 1964) (unfortunately out of print)

Michael Kimber – July 2020