

Melodic and Harmonic Intonation in Relation to Equal Temperament

When playing melodically we are influenced by the compelling relationships among perfect intervals—fourths, fifths, and octaves. When tuning chords and double stops we are influenced by the harmonic series, including harmonically pure thirds and sixths. When playing with keyboard accompaniment we are influenced by equal temperament. To play in tune we must weave an artistic path among these influences, listening intently and adjusting to achieve the most musically satisfying effect at any given moment.

1 • MELODIC INTONATION

Harmonically Pure Perfect Fifths
(Basis of Pythagorean Intonation)

compare:
 $5.0625 \times C \neq 5 \times C$
 $E \neq E-$
 The difference is 21.5 cents*
 — the *syntonic comma*.

2 • HARMONIC INTONATION

The Harmonic Series
(Basis of Just Intonation)

Every note we play is actually a series of harmonics extending even beyond the pitches shown here. When we tune harmonic intervals, as in double stops and chords, it is agreement among harmonics that informs us that the notes are in tune with each other.

Pythagorean E is higher than the E- of the harmonic series of C. While melodically desirable, E is not in tune harmonically with the C (or G). Adjustment is needed when both E and C (or G) are sounding at the same time. We can observe this by playing the two double stops at right. The E must be adjusted!

Pythagorean Tuning Extended to Yield
the Seven Notes of the C Major Scale

The same notes within one octave

These pitches, in stepwise order, form the Pythagorean C major scale below.

The Primary Triads (I, IV, V) in C Major
Tuned Harmonically (Just Intonation)

The roots of the primary triads are pure perfect fifths apart. The third and fifth of a just triad are tuned in relation to the harmonic partials of its root.

Note that the third of the just triad is a comma lower than if it had been derived by tuning only perfect fifths (Pythagorean).

IV I V

≡ = partials that coincide when tuning pure fifths

The Pythagorean C Major Scale (compare just scale)

The numbers indicate the size in cents* of each step.

Notice consistently wide whole tones and narrow semitones, reflecting our “expressive” melodic tendencies.

The same notes within one octave

The pitches of these primary triads provide the notes of the just intonation C major scale below.

IV I V

Pythagorean Tuning — Overlapping “Circle” of Fifths

The numbers indicate the frequency ratio of each fifth or fourth.

$A\flat \neq G\sharp$

The \flat/\sharp difference is 23.5 cents* — the *Pythagorean comma*.

The Pythagorean comma reflects our melodic tendencies to play sharps higher than flats and to emphasize major/minor distinctions. (Harmonically we must often do the reverse.)

The Just Intonation C Major Scale (compare Pythag. scale)

The numbers indicate the size in cents* of each step.

Note that the interval from D to A- is a comma narrower than a true fifth, requiring adjustment when both notes are sounding at the same time. Most players find the 182-cent whole tones and 112-cent semitones melodically less satisfying than their Pythagorean counterparts.

3 • EQUAL TEMPERAMENT

Equal temperament is the present-day [mis]tuning of keyboard instruments. It avoids the Pythagorean comma by using fifths that are about 2 cents (1/12 comma) narrow. Unlike historic keyboard temperaments, it makes no attempt to provide any harmonically pure thirds. Equal temperament, with its 12 equal semitones, only approximates true intervals, although it does so equally in all keys. **Good intonation is better!**

* There are 1200 cents in an octave — 100 cents in each semitone on the piano.