Probability of Banks Exit in an Economy à la Diamond-Dybvig

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Summary

This paper provides a simple example of an economy in which differences between investors' and bankers' beliefs about expected return of the long-term asset exhibit may be the most important component of the probability of banks exit. Other factors with influence on this probability are the risk of liquidity shocks on investors, maximum long-term return, intertemporal discount rate and investors’ rate of relative risk-aversion.

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I. Introduction

This paper provides a simple example of an economy in which differences between investors' and bankers' beliefs about expected return of the long-term asset exhibit may be the most important component of the probability of banks exit.

I follow Diamond and Dybvig (1983) and Jacklin (1987) in viewing investors as ex-ante identical individuals whom lives only three periods. During the first period, their types (patient or impatient) are unknown to themselves but starting second period each investor realizes his own type. My investor's preferences are a hybrid between Diamond and Dybvig's extreme case and the more general case exhibit by Jacklin. As result, patient investors receive utility from second and third period consumption and impatient investors do not perceive utility from third period consumption.

In my example, production technology is essentially a variant of those shown by Jacklin and Bhattacharya (1989): If production process is interrupted before the beginning of the third period, the investor obtains a low riskless return. But if production process is not interrupted then it yields a risky return whose objective probabilities are unknown.

My model differs from those of Jacklin and Bhattacharya in four important aspects. First, they assume that output from liquidation can be hold until the third period. In contrast, I assume that this supplementary storage technology is not available and early liquidation of long-term investment is impossible. Second, they consider that investors' beliefs about expected long-term return are asymmetric, whereas I assume that these beliefs are identical among investors. Third, random shocks are aggregate in the sense described by Wallace (1988). Fourth, I consider infinitely lived agents called bankers who have access to the same technology and whose role is to provide a higher riskless asset to the investors. It is possible because bankers are risk neutral and are able to obtain the expected return during consecutive generations of investors.

Both features, preferences and technology, are essential to show that autarkic solution equals socially optimal risk sharing, so there is not a role for bankers as liquidity providers, but only as providers of a riskless long-term asset. In addition, if investors and bankers have different beliefs about expected return of the long-term asset, banks would initiate a "war of attrition" as defined by Fudenberg and Tirole (1986). The rationale is as follows: If bankers and investors have the same beliefs, Bertrand equilibrium[1] arises and banks offer the contract that maximizes investors' welfare. On the other hand, if bankers and investors have different beliefs, offering the same contract would generate losses. Nevertheless, bankers will continue offering this contract because they believe others would to stop first, so each bank would eventually earn monopolistic gains. This story constitutes an example of the symmetric and stationary mixed-strategy equilibrium, which Fudenberg and Tirole called Perpetual Selection.

In this kind of equilibrium, banks have an estimable probability of exit. In particular, this paper finds an expression for this probability. Bankers' and investors' beliefs about long-term return, investors' risk aversion, and the probabilities of a liquidity shock are found as main arguments of this function.

The balance of the paper is organized as follows: Section II describes autarkic and optimal solution without bankers. Section III considers the participation of banks when they are a monopoly or an oligopoly according to Bertrand conjectures. Section IV analyzes the war of attrition equilibrium and calculates bank exit probability. Finally, section V concludes the paper.

[1] Adao and Temzelides (1998) have the first analysis of a Bertrand Equilibrium in a Diamond and Dybvig Economy, but their work is focused on depositor beliefs on a eventual bank run.

II. Economy without Bankers

i. Preferences. This section of the paper describes a simple economy of infinite and consecutive generations of investors who live only three periods. Investors are a continuum of agents with total measure one. In T=t-1, investors are identical and do neither know their own nor the others'. In T=t each investor learns his own type, but does not learn about the type of others. Either he is patient and lives until the end of period T=t+1 or impatient and lives only until the end of period T=t. They know that an investor is impatient with probability π and patient with probability 1-π. Preferences for consumption in periods 1 and 2 are represented by:

$$u(c_t, c_{t+1}) = \begin{cases} u(c_t) & \textit{if individual is impatient} \\ u(c_t) + r\,u(c_{t+1}) & \textit{if individual is patient} \end{cases}$$

where u: $R_{++} \rightarrow R$ is twice continuously differentiable, increasing, and strictly concave, u(0) = 0 and satisfies Inada conditions $u'(0) = \infty$ and $u'(\infty) = 0$.

ii. Endowments and technologies. At T=t-1, all investors receive capital good endowments which when invested yields a return in the form of the consumption good. There are two available technologies: A short-term technology that yields one unit of consumption good for each unit of capital good, and a long-term technology which produces a random return. Capital good is infinitely divisible, whatever portion of the production can be invested in the short-term technology, and the remaining will be automatically invested in the long-term technology. However, when investment decisions are made in period T=t-1, no changes are feasible which means that long-term capital investments are irreversible.

The random returns constitute an infinite sequence S whose components could be R or zero each three periods. Now, I borrow the reasoning from clever actors invented by Kreps (1983) in order to understand the formation of subjective probabilities (beliefs) about that random return.

Let $X_t(S) = 1$ if the t^{th} component of S is R, and $X_t(S) = 0$ if the t^{th} component of S is 0.

I assume that this sequence S has a powerful property called exchangeability, which means any permutation of the outcomes or components of the sequence has the same joint distribution.

With this assumption, it is feasible to define a random variable equal to the limiting empirically observed frequency of high returns each generation

$$\lim_{T\to\infty} \frac{\sum_{t=1}^{T} X_t}{T/3} \equiv \tilde{h}$$

Investors have identical beliefs about the value of this limit. This beliefs are represented by a probability distribution function F:[0,1] → [0,1], which means that investors subjectively assess the probability distribution function F for the limiting empirical frequency. Using the exchangeability property of S, De Finetti´s theorem guarantees that the expected value of $\tilde{h}$ equals the probability of higher returns during a generation of investors born in a given period T=t-1

$$\Pr ob\left(\tilde{R}_{t+1} = R\right) = \Pr ob\left(X_{t+1} = 1\right) = \int_0^1 \tilde{h}\, dF(\tilde{h}) = E(\tilde{h}; F) \equiv h$$

iii. Autarky. In the simplest case, in which there is no trade among investors, each investor chooses his consumption profile. The representative investor maximizes his expected utility and solving the following constrained optimization problem in period T=t-1:

$$\begin{aligned} &\underset{c_t, c_{t+1}}{Max} \quad p\, u(c_t) + (1-p)\, u(c_t) + h(1-p)\, r\, u(c_{t+1}) \\ &\textit{subject to} \\ &\qquad\qquad c_{t+1} = R(1-c_t) \end{aligned}$$

The autarkic allocation satisfies the first order condition:

$$u'(c_t) = E(\tilde{R}; h)\,(1-p)\, r\, u\left(R(1-c_t)\right)$$

where $E(\tilde{R}, h)$ is defined as hR.

iv. Optimal risk sharing without bankers. If I only considers investors, there will be an unique symmetric Pareto optimal allocation (c_t,c_{t+1}), obtained by solving

$$\underset{c_{1,t},c_{2,t},c_{2,t+1}}{Max} \quad p\,u(c_{1,t})+(1-p)\,u(c_{2,t})+h(1-p)\,r\,u(c_{2,t+1})$$

subject to

$$c_{2,t+1}=R\big(1-p\,c_{1,t}-(1-p)\,c_{2,t+1}\big)$$

Note that, in this case I only consider two consumption bundles in t, one for each type of investors ($c_{1,t}$ represents inpatient's consumption and $c_{2,t+1}$ represent patient consumption). The resulting optimal allocation satisfies the first-order condition:

$$u'(c^*_{1,t})=u'(c^*_{2,t})=E(\tilde{R};h)\,(1-p)\,r\,u\big(R\big(1-p\;c^*_{1,t}-(1-p)\;c^*_{2,t}\big)\big)$$

The first equality indicates that $c^*_{1,t} = c^*_{2,t}$. When this result is replaced on the second equality, the autarkic solution is obtained. Therefore, the autarkic solution is Pareto optimal when bankers are excluded.

III. Economy with bankers

In the last section I showed that, using a variant of the Diamond-Dybvig`s model, autarkic solution can achieve the Pareto optimal allocation and demand deposits are not necessary. This is not generally true. In this section I show that, with neutral-risk infinite-lived bankers, deposits do improve investors` welfare respect to results from autarky.

i. One banker. In an attempt to model banks as profit-maximizing institutions rather than a social planner, I assume that there is a monopolist who is called banker and whose unique objective is to maximize profits. In addition, I consider that banker lives infinite periods, so he faces a new set of investors each three periods. The banker does not receive any endowment but does have access to long-term and short-term technologies. There is an infinite number of potential investors each three periods.

Banker offers a contract $\{d_t,d_{t+1}\}$ which requires an investment of one unit of the capital good in exchange of the right to withdraw either d_t in period t or d_{t+1} in period t+1**.** When an investor accepts this contract, his expected utility is given by

$$u(d_t)+r\,(1-p\,)\,u(d_{t+1})$$

On the other hand, the expected per generation profit earned by the banker is defined here as the excess of return of a unit of investment over cost of the bundle offered to the investors

$$E\left(\tilde{R};h\right)(1-d_t)-\left(1-p^{\,0}\right)(d_{t+1})$$

Two observations about the monopolist's rationality are of interest. The first one is whether or not each investor is willing to acquire the bundle offered by the banker. This requirement or participation constraint is satisfied if investors prefer this bundle rather than the autarkic allocation. The second observation is that the banker's beliefs about long-term return are identical to those of investors. Therefore, subject to the participation constraint, the allocation that maximize expected profits is obtained by solving the following minimization problem:

$$\begin{aligned} &\min_{d_t,d_{t+1}} \quad E\left(\tilde{R};h\right)d_t+(1-p\,)d_{t+1} \\ &s.a. \quad u(d_t)+r\,(1-p\,)\,u(d_{t+1})\geq u\left(c_t^*\right)+r\,h(1-p\,)\,u\left(c_{t+1}^*\right) \end{aligned}$$

ii. Several Bankers. In this section, I present the case in which there are J bankers in the economy. All of them have identical beliefs about the long-term asset return and everyone has access to the same technology. Each banker offers a sequence of contracts every three periods $\{d_t,d_{t+1}\}$. On the other hand, a given member of a generation of investors searchs the bank that offers a contract which maximizes his utility. When this investor finds that bank, he puts all his capital in that bank and the rest receives no financing.

The expected profit of a banker j who offers a contract $\{d_t, d_{t+1}\} >> \{d_t, d_{t+1}\}_{-j}$ is given by the excess of return of an unitary investment over cost of the bundle offered to the investors

$$D_j = (1 - d_t^j) E(\tilde{R}; h) - (1 - p) d_{t+1}^j$$

However, if a banker offers a contract as $\{d_t, d_{t+1}\} << \{d_t, d_{t+1}\}_{-j}$ then his profits are zero. Finally, if all bankers offers the same contract $\{d_t, d_{t+1}\} = \{d_t, d_{t+1}\}_{-j}$ then their profits are D_j/J[2].

In this context, a Bertrand equilibrium is defined as both a pair of contracts ($\{d_t, d_{t+1}\}_j, \{d_t, d_{t+1}\}_{-j}$) and a pair of investment amounts (k_j,k_{-j}) such that

a) If $\{d_t, d_{t+1}\} << \{d_t, d_{t+1}\}_{-j}$ then $k_j = 0$, $k_{-j} = D_{-j}/J-1$
b) If $\{d_t, d_{t+1}\} >> \{d_t, d_{t+1}\}_{-j}$ then $k_j = D_j$, $k_{-j} = 0$,
c) If $\{d_t, d_{t+1}\} = \{d_t, d_{t+1}\}_{-j}$ then $k_j + k_{-j} = D_j$.
d) All bankers are not willing to modify previous situation.

Therefore, the unique equilibrium contract is $\{d_t, d_{t+1}\} = \{d_t, d_{t+1}\}_{-j} = \{d_t^*, d_{t+1}^*\}$ which solves the following problem:

$$\begin{aligned}
&\underset{d_t, d_{t+1}}{Max} \quad u(d_t) + (1-p) r\, u(d_{t+1}) \\
&s.t. \\
&\qquad\qquad d_{t+1} = E(\tilde{R}; h)(1 - d_t)
\end{aligned}$$

This solution is a Bertrand Equilibrium because it is the unique bundle such that if a banker offers a better contract then he incurs in losses, but if he reduces his offer then the investor abandons him by any other.

[2] Remember that this is an arbitrary division. In fact, whatever else criterion could be used.

iii. Example to obtain the viability condition for banking. In this subsection, I offer a parameterized example of the model described in Section II and III. For this example, I show that cost of the bundle offered by banks is a function of investors' welfare in autarky. This result is important to reveal which assumptions support the viability of banking.

The only additional assumption is that $u(x)=x^{\alpha}$, where $\alpha < 1$. Solving the maximization of the autarkic case, I get an expression for investors' welfare:

$$U_{\alpha} = \left(1+\frac{1}{R}\right)\left(c_t^*\right)^{\alpha} = \frac{\left(1+\frac{1}{R}\right)}{\left[1+\left(h\left(1-\pi\right)\rho\right)^{\frac{1}{1-\alpha}}\left(R\right)^{\frac{\alpha}{1-\alpha}}\right]^{\alpha}}$$

Thus U_a is an increasing function in π and a decreasing function in h, ρ and R.

The cost of the bundle offered by the banker to the investors is obtained by solving the minimization problem of the monopolist:

$$C_m = q\,U_{\alpha}^{\frac{1}{\alpha}} = q\left(1+\tfrac{1}{R}\right)^{\frac{1}{\alpha}} c_t^*$$

where

$$0 < q \equiv \frac{\left[E(\tilde{R};h)^{\frac{\alpha}{\alpha-1}} + \left(1-\pi\right)\rho^{\frac{2-\alpha}{1-\alpha}}\right]}{\left[E(\tilde{R};h)^{\frac{\alpha}{\alpha-1}} + \left(1-\pi\right)\rho^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\alpha}}} < 1$$

Thus C_m is an increasing function in π and a decreasing function in h, ρ and R.

Using these definitions is feasible to determine the unique necessary and sufficient condition, in terms of the parameters of the model, for the viability of banking.

In this example, banking is viable if and only if $E\left(\tilde{R};h\right) > q\,U_{\alpha}^{\frac{1}{\alpha}}$. Rationale is as follows: When this condition is satisfied, monopolist obtains positive expected profits and investors lend to the banker. However, if this condition is not satisfied, the banker who offers an attractive bundle incurs in losses.

Particularly, if $\alpha < 1$, $E(\tilde{R}; h) > 1$ and investors' and bankers' beliefs are identical then the viability for banking is guaranteed in this example.

IV. War of Attrition among Bankers

In this section, I discuss some of the difficulties that may arise when I attempt to differentiate beliefs between investors and bankers. Now, suppose that bankers do not know whether F is the investors' subjective probability distribution or it is not. F would be the right distribution with a probability p, but it is also feasible that right distribution would be F' ≠ F with a probability 1-p. As result, it is feasible obtain a new belief about limiting empirically observed frequency of high returns

$$\Pr ob(\tilde{R}_{t+1} = R) = \Pr ob(X_{t+1} = 1) = \int_0^1 \tilde{h}\, dF'(\tilde{h}) = E(\tilde{h}; F') \equiv h'$$

Uncertainty about truly distribution of investors' subjective probabilities has an important consequence on banking viability: If difference between bankers' expected long-term return and investors' expected long-term return is negative, banking activities could be not viable.

i. Implications on Bertrand Equilibrium. In the case of the Bertrand equilibrium among J bankers, all of them must offer an attractive bundle to investors (i.e. a bundle with a utility more than autarky's welfare). Thus, with probability p, they have to offer the same contract $\{d_t^*, d_{t+1}^*\}$ described in section III; however with probability 1-p, they must offer a contract $\{d_t', d_{t+1}'\}$ that consider h' rather than h.

By construction, the contract $\{d_t^*, d_{t+1}^*\}$ does not origin losses or gains to bankers. However, the new contract $\{d_t', d_{t+1}'\}$ produces a loss equal to the difference between expected long-term return estimated with h and those estimated using h'.

According to this, the expected present value of bankers' profit until period τ will be negative:

$$V(J,t-1)=\sum_{t=0}^{t-1}\frac{r^{t}(1-p)}{J}\left\{E\left(\tilde{R};h\right)-E\left(\tilde{R};h'\right)\right\}$$

However, if I consider the case in which a given banker will become a monopolist in period τ, then the expected present value of bankers equals to:

$$V(1,t)=\sum_{t=0}^{t-1}\frac{r^{t}(1-p)}{J}\left\{E\left(\tilde{R};h\right)-E\left(\tilde{R};h'\right)\right\}+\sum_{t=t}^{\infty}r^{t}\left\{p\left[E\left(\tilde{R};h\right)-q\,U_{a}^{\frac{1}{a}}\right]+(1-p)\left[E\left(\tilde{R};h\right)-E\left(\tilde{R};h'\right)\right]\right\}$$

ii. Stationary, Symmetric and Mixed-Strategy Equilibrium. In this section, I introduce the possibility that bankers could stop their activities in some period. It happens because I suppose that they follow a mixed-strategy in which a banker stop at t, with a probability q, if the others bankers have not stopped before then. Probability of continuation is 1-q.

This story reminds a war of attrition among bankers in which they remain in competition, even they are incurring in losses, because they believe that other bankers would does drop out before then, so each bank would eventually becomes a monopolist and obtains positive gains. Fudenberg and Tirole (1986) named this kind of competence "Perpetual Selection".

I define an unique Nash, stationary, and symmetric Equilibrium, which is given by the mixed strategy that defines an exit probability q_t = q, conditional to that opponents have not stopped previously. This probability is such that

$$V(J,t-1)=qV(1,t)+(1-q)V(J,t)$$

Replacing expressions V(1) and V(J), I obtain a formulation for $q_t = q$, which is given by:

$$q = \frac{\left(\frac{p}{J}\right)\left(E\left(\tilde{R};h\right) - E\left(\tilde{R};h'\right)\right)}{\left(\frac{p}{J}\right)\left(E\left(\tilde{R};h\right) - E\left(\tilde{R};h'\right)\right) + \frac{p\left(E\left(\tilde{R};h\right) - q\,U_a^{\frac{1}{a}}\right) + (1-p)\left(E\left(\tilde{R},h\right) - E\left(\tilde{R};h'\right)\right)}{1-r}}$$

This is important to review some conclusions from this result. First, difference between investors' and bankers' beliefs about expected return of the long-term asset exhibit affect positively the probability of banks exit. Other factors with influence on this probability are the risk of liquidity shocks on investors, maximum long-term return, intertemporal discount rate and investors’ rate of relative risk-aversion. As q is an increasing function in U_a, then q is an increasing function in π and a decreasing function in h, ρ and R. Also it is important to precise that a higher probability of that beliefs differ between investors an bankers, p, implies a higher probability of bank exit.

V. Conclusions

This paper uses a simple example in order to offer a theoretic analysis of how investors’ beliefs on long-term return would affect the behavior strategies follow by bankers. More specifically, this work finds a formulation for exit probability of banks. This model establishes the necessary and sufficient conditions to guarantee banking viability in a economy with infinite lived bankers and investors ala Diamond and Dybvig. This analysis also suggest the existence of a unique Nash, symmetric and stationary equilibrium in which all bankers continues competition with a implicit probability of exit, so each bank would eventually earns monopolistic profits. Finally, this paper reveals that the main determinant of this probability is the difference between investors’ and bankers’ beliefs about expected value of long-term return.

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