Modern filter theory is based on many diverse but interrelated discoveries over the past two centuries. In 1785 Laplace was writing the almost modern Laplace transform to solve certain differential equations with variable coefficients. Baron Jean-Baptiste-Joseph Fourier, as a prefect in Napoleon's army, invented a method for solving equations that described the conduction of heat in solid bodies and published it in 1807. Laplace was intrigued and obtained additional results that in turn inspired Fourier's discovery of his own transform. Many 19th century mathematicians did not accept Fourier's claim that any initial temperature distribution could be decomposed into a simple arithmetic sum consisting of a fundamental variation and higher frequency harmonics, and the theoretical basis of the Fourier transform was not formally published until 1822.

In 1829, Carl Gustav Jacob Jacobi derived and published the main properties of a set of new functions useful for solving elliptic integrals. At that time, the elliptic functions were used to solve problems in finding the length of an arc of an ellipse and in determining the motion of a pendulum.

The first known record of sampling analog signals is found on pages 420-425 of Treatise on Electricity and Magnetism (1873) by James Clerk Maxwell in his discussion on the equivalent resistance of a periodically switched capacitor.

Filter technology was officially born in 1915 when K. Wagner (Germany) and G. Campbell (United States), working independently, proposed the basic concept of the filter. Their results evolved from earlier work on loaded transmission lines and the classical theory of vibrating systems.

In 1923, Zobel published a practical method of designing image parameter filters with an unlimited number of reactances. This filter method is a little used technique today because it gives the designer only a very loose control over the passband and stopband characteristics. However, it was the only known method until 1939 and the only practical method until computers became widespread in the mid-1950s. S. Darlington in the United States and W. Cauer in Germany, both inspired by the work of Norton, published a theory in 1939 that involved a set of problems relating to modern synthesis procedure. The filters proposed by Darlington and Cauer used the Jacobian elliptic integral to determine element values for very sharp rolloff filters with Tchebyscheff (or equiripple) behavior in both the passband and the stopband.

This behavior takes its name from the Russian mathematician who in the last century developed a powerful method of approximation by means of orthogonal polynomials.
The elliptic filters of Darlington and Cauer should not be confused with Tchebyscheff filters since the latter exhibit equiripple behavior in either the passband or the stopband, but not both. The importance of polynomial filter synthesis was not recognized immediately because of the

extremely heavy burden on computation required. It was not until the mid-1950s that Cauer-Darlington (elliptic) filters came into widespread use.

The theory of sampling analog signals was also well developed by the late 1950s. In the 1960s several schemes were proposed that used switches and capacitors to simulate resistors in filters. It was shown that the filter transmittances had the important property of depending only on capacitor ratios.

The Fairchild uA 709 was introduced in 1965 at $70.00 and became the first integrated circuit (IC) operational amplifier (op-amp) manufactured with such high yields in volume production that it facilitated the implementation of active RC filters equaling in performance and usefulness its passive counterpart. Although the op-amp was a major component of the filter circuit and eliminated the need for bulky inductors, discrete resistors and capacitors were still required because IC technology did not permit resistors and capacitors to be integrated with a high degree of accuracy.

That same year, James W. Cooley of IBM's Thomas J. Watson Research Center and John W. Tukey of the Bell Telephone Laboratories in Murray Hill, N.J. rediscovered a program known as the fast Fourier transform (FFT) which allowed a speed increase of up to several orders of magnitude in the calculation of the Fourier transform. The FFT has become the basis for the implementation of many digital filters since the computations can be done in real time by high speed digital signal processor (DSP) chips.

The importance and significance of many of the ideas proposed for analog sampled data signal processing had to wait until technology provided the means of turning ideas into practical reality. In 1972, it was suggested that Metal Oxide Semiconductor (MOS) technology would be applicable for the construction of integrated circuit analog sampled data filters using switched capacitors to simulate resistors in active filters. This suggestion was then followed by a rapid development in the implementation of analog signal processing integrated circuits. The switched capacitor techniques provided very high accuracy in the design of analog IC filters since the filter parameters depended only on the ratios of capacitors and this ratio can be controlled to a very high accuracy in the design of MOS integrated circuits.